

Graph each function. State the domain, range and asymptote.

Note: Your graph should include at least three clearly labeled points and the asymptote.

1. $g(x) = 5(2^{-x})$
 vertical stretch by 5
 reflected in y-axis
 H.A. $y=0$

x	2^x
0	1
1	2
2	4

x	$5(2^{-x})$
0	5
-1	10
-2	20

2. $g(x) = 5^{\frac{x}{4}}$ H.A. $y=0$
 horizontal stretch by 4

x	5^x
0	1
1	5
2	25

x	$5^{\frac{x}{4}}$
0	1
4	5
8	25

3. $g(x) = \log(x+5)$
 shifted left 5
 VA $x=-5$

x	$\log x$
1/10	-1
1	0
10	1
100	2

x	$g(x)$
-4 1/10	-1
-4	0
5	1
95	2

4. $g(x) = 3 - \ln x$
 reflection in x
 up 3
 VA $x=0$

x	$\ln x$
1/e	-1
1	0
e	1
e^2	2

x	$g(x)$
1/e	4
1	3
e	2
e^2	1

5. Given the set of transformations on f , $f(x) = \log_4 x$, write the equation that yields g .

a. 3 units left, 2 units up

$$g(x) = \log_4(x+3) + 2$$

b. 4 units right, reflection in the x-axis

$$g(x) = -\log_4(x-4)$$

c. reflection in the y-axis, down 3

$$g(x) = \log_4(-x) - 3$$

6. Write the transformed function.

The function $f(x) = 8 \cdot 7^{2x} - 5$ is horizontally stretched by a factor of 2, vertically compressed by a factor of 0.5, translated 1 unit right, and reflected across the x-axis.

$$f\left(\frac{1}{2}x\right) = 8 \cdot 7^x - 5$$

$$= 4 \cdot 7^x - 5/2$$

$$g(x) = -4 \cdot 7^{x-1} + 5/2$$

In 7-14, evaluate each expression WITHOUT A CALCULATOR.

7. $\log_3 81^5$

$5 \log_3 81 = 5(4) = 20$

11. $\ln e = 1$

8. $\log_{\frac{1}{4}} 8 = \frac{\log_2 8}{\log_2 \frac{1}{4}} = \frac{3}{-2}$

12. $2 \log 5 + \log 4$

$\log 5^2 + \log 4$
 $\log(100) = 2$

15. Rewrite $\log_{16} \frac{1}{4} = -\frac{1}{2}$ in exponential form.

$16^{-1/2} = \frac{1}{4}$

9. $(\log 10^{8x})(\ln e^7) = (8x)(7) = 56x$

13. $e^{\ln 3xy^2}$

$3xy^2$

16. Rewrite $3^{-4} = \frac{1}{81}$ in logarithmic form.

$\log_3 \frac{1}{81} = -4$

10. $5^{\log_5 30 - \log_5 2} = 5^{\log_5 15} = 15$

14. $\log_6 \frac{1}{216} = -3$

17. Colleen's station wagon is depreciating at a rate of 9% per year. She paid \$24,500 for it in 2002. What will the car be worth in 2008 to the nearest hundred dollars?

$V(t) = 24,500(0.91)^t$
 $V(6) = 24,500(0.91)^6 = \$13,900$

18. A parcel of land Jason bought in 2000 for \$100,000 is appreciating in value at a rate of about 4% each year. Write a function to model the appreciation of the value of the land, and determine (algebraically) in what year will the land double its value?

$A = 100000(1.04)^t$
 $200000 = 100000(1.04)^t$
 $2 = 1.04^t$
 $\log_{1.04} 2 = t$

$t = \frac{\log 2}{\log 1.04} = 17.67$

by 2018 the land will have doubled its value
 $A = Pe^{rt}$

19. A deposit of \$10,000 is made in a savings account for which the interest is compounded continuously. The balance will double in 5 years.

a. What is the annual interest rate for this account?

13.86%

$20000 = 10000e^{r(5)}$
 $2 = e^{5r}$
 $r = \frac{\ln 2}{5} = 0.1386$

b. Find the balance after 3 years.

$A = 10000e^{0.1386(3)} = \$15,155.83$

20. Ariana has a choice of two investments. She can invest \$12,000 at 5% for 8 years, or she can invest \$9000 at 6.5% for 7 years. Both accounts are compounded continuously. Which investment will result in the greater amount of interest earned?

The first one (\$12000 @ 5%)

$A = Pe^{rt}$
 $A = 12000e^{0.05(8)}$
 $\$17,901.89$
 $\$5,901.89$
 interest earned

$A = 9000e^{0.065(7)}$
 $A = \$14,185.56$
 $\$5185.56$
 interest earned

Use the natural decay function, $N(t) = N_0 e^{-kt}$, to find the decay constant for a substance that has a half-life of 1000 years.

$$\frac{1}{2} = e^{-k(1000)}$$

$$k = \frac{\ln 1/2}{-1000} = \boxed{0.000693}$$

22. Use the natural decay function, $N(t) = N_0 e^{-kt}$, to find the age of a fossil containing 35% of the original amount of a particular substance. This substance has a half-life of 2450 years.

$$\frac{1}{2} = e^{-k(2450)}$$

$$\ln \frac{1}{2} = -2450k$$

$$k = \frac{\ln 1/2}{-2450} = \boxed{0.000283}$$

$$0.35 = e^{-0.000283t}$$

$$t = \frac{\ln 0.35}{-0.000283} = \boxed{3709.62 \text{ years}}$$

23. Newton's Law of Cooling: $T = T_s + (T_0 - T_s)e^{-kt}$, where T_0 is the initial temperature and T_s is the surrounding temperature.

T_0 Your car just overheated on the drive home from work and is stuck on the side of the road. It overheated at 300°F and can be driven again at 230°F . If $k = 0.0048$ and it is 65°F outside, how long (in minutes) do you have to wait until you can continue driving?

$$230 = 65 + (300 - 65)e^{-0.0048t}$$

$$165 = 235e^{-0.0048t}$$

$$\frac{33}{47} = e^{-0.0048t}$$

$$\frac{\ln \frac{33}{47}}{-0.0048} = \boxed{t = 73.68 \text{ minutes}}$$

Use the change of base formula to evaluate:

$$24. \log_5 7 = \frac{\log 7}{\log 5} = 1.209$$

$$25. \log_{\frac{1}{3}} \frac{1}{5} = \frac{\log \frac{1}{5}}{\log \frac{1}{3}} = 1.465$$

Use $\log_a 2 \approx 0.3562$ and $\log_a 3 \approx 0.5646$ to rewrite and evaluate the following expressions.

$$26. \log_a \left(\frac{2}{3}\right) \approx -0.2084$$

$$\log_a 2 - \log_a 3$$

$$0.3562 - 0.5646$$

$$27. \log_a 6 = \log_a (2 \cdot 3)$$

$$= \log_a 2 + \log_a 3$$

$$= 0.3562 + 0.5646$$

$$\boxed{0.9208}$$

$$28. \log_a \frac{9}{4} = \log_a \frac{3^2}{2^2}$$

$$\log_a 3^2 - \log_a 2^2$$

$$2 \log_a 3 - 2 \log_a 2$$

$$2(0.5646) - 2(0.3562) = \boxed{0.4168}$$

Expand each expression.

$$29. \log_5 7x^3y$$

$$\log_5 7 + 3 \log_5 x + \log_5 y$$

$$30. \ln \left(\frac{x^2 y^3}{x-y}\right)$$

$$2 \ln x + 3 \ln y - \ln(x-y)$$

$$31. \ln \sqrt{x^3 y^2} = \ln (x^3 y^2)^{1/2}$$

$$\frac{3}{2} \ln x + \ln y$$

Condense each expression.

32. $\frac{1}{3} \log_4(x+y)$

$\log_4(x+y)^{1/3}$
 $\log_4 \sqrt[3]{x+y}$

33. $3 \ln(x-2) - 2 \ln(x+2)$

$\ln \frac{(x-2)^3}{(x+2)^2}$

34. $\log 8 + 3 \log x - \log 7$

$\log \frac{8x^3}{7}$

Solve each equation algebraically. Work MUST be shown.

35. $16^{3x} = 8^{x+6}$

$(2^4)^{3x} = (2^3)^{x+6}$

$12x = 3x + 18$

$9x = 18$

$x = 2$

36. $-4 \log_6(9x) - 7 = -23$

$-4 \log_6(9x) = -16$

$\log_6(9x) = 4$

$6^4 = 9x$

$x = \frac{6^4}{9} = 144$

37. $12^{x-1} = 20^2$

$\log_{12} 12^{x-1} = \log_{12} 400$

$x-1 = \frac{\log 400}{\log 12}$

$x = \frac{\log 400}{\log 12} + 1 = 3.411$

38. $\left(\frac{1}{16}\right)^{x+5} = 8^2$

$(2^{-4})^{x+5} = (2^3)^2$

$-4x - 20 = 6$

$-4x = 26$

$x = -13/2$

39. $216^{x/3} = 36^{2x+3}$

$(6^3)^{x/3} = (6^2)^{2x+3}$

$x = 4x + 6$

$-3x = 6$

$x = -2$

40. $7 \cdot 9^{2x-4} + 3 = 45$

$7 \cdot 9^{2x-4} = 42$

$9^{2x-4} = 6$

$\log_9 9^{2x-4} = \log_9 6$

$2x-4 = \frac{\log 6}{\log 9}$

$x = \frac{\log 6}{\log 9} + 4$

$x = 2.408$

41. $e^{4x} - 7 = 10$

$e^{4x} = 17$

$\ln e^{4x} = \ln 17$

$4x = \ln 17$

$x = \frac{\ln 17}{4} = 0.708$

42. $3 + e^{-2x} = 11$

$e^{-2x} = 8$

$-2x = \ln 8$

$x = \frac{\ln 8}{-2} = -1.040$

43. $\log_5(4x-5)^2 = 6$

$\sqrt{5^6} = \sqrt{(4x-5)^2}$

$4x-5 = \pm 5^3$

$4x = 5 \pm 5^3$

$x = \frac{5 \pm 5^3}{4}$

32.5
 -30

44. $\log x - \log 8 = 3$

$\log \frac{x}{8} = 3$

$10^3 = \frac{x}{8}$

$x = 8 \cdot 10^3 = 8000$

45. $\ln(x^2 - 9) = \ln(5x + 5)$

$x^2 - 9 = 5x + 5$

$x^2 - 5x - 14 = 0$

$(x-7)(x+2) = 0$

$x = 7$

46. $\log x^3 + \log 8 = 3$

$\log 8x^3 = 3$

$10^3 = 8x^3$

$x = \sqrt[3]{\frac{10^3}{8}} = 5$

47. $\log(x^2 - 1) - \log 12 = 1$

$\log \frac{x^2 - 1}{12} = 1$

$10^1 = \frac{x^2 - 1}{12}$

$120 = x^2 - 1$

$x^2 = 121$
 $x = \pm 11$

48. $\ln 5x - 9 = 11$

$\ln 5x = 20$

$5x = e^{20}$

$x = \frac{e^{20}}{5} = 97033039.08$