

Chapter 3 Review

ANSWER KEY

1. $(x^3 + x^2) \div (x - 1)$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & 0 & \\ & & 1 & 2 & 2 \\ \hline & 1 & 2 & 2 & 2 \end{array}$$

of strips is $x^2 + 2x + 2 + 2/x - 1$

2. $T(x) = N(x) \cdot C(x)$
 $= (0.3x^2 - 1.6x + 14)(-0.001x^2 - 0.06x + 8.3)$
 $= -0.0003x^4 - 0.018x^3 + 2.49x^2$
 $+ 0.0016x^3 + 0.096x^2 - 13.28x$
 $- 0.014x^2 - 0.84x + 116.2$

$T(x) = -0.0003x^4 - 0.0164x^3 + 2.572x^2 - 14.12x + 116.2$

3. refer to page 186 #23 for a similar problem, if needed

$$V(x) = (11 - 2x)(14 - 2x)(x)$$

$$= (154 - 50x + 4x^2)(x)$$

$V(x) = 4x^3 - 50x^2 + 154x$

a) maximum volume = 140.04 in.³
 length of side of square = 2.04 in. } use graphing calculator to find maximum

4. x : length of box

$$V = x(x - 2)(x + 5)$$

$$V = x(x^2 + 3x - 10)$$

$$V = x^3 + 3x^2 - 10x$$

$$x^3 + 3x^2 - 10x = 24$$

$$x^3 + 3x^2 - 10x - 24 = 0$$

$$\begin{array}{r|rrrr} 3 & 1 & 3 & -10 & -24 \\ & & 3 & 18 & 24 \\ \hline & 1 & 6 & 8 & 0 \end{array}$$

$$x^2 + 6x + 8 = 0$$

$$(x + 2)(x + 4) = 0$$

$x = 3$ is ~~2~~, A
 length is 3 inches

$$5. \quad V_{\text{ROCKET BODY}} = V_{\text{CYLINDER}} + V_{\text{CONE}}$$

$$= \pi r^2(60) + \frac{1}{3}\pi r^2(2r)$$

$$558\pi = 60\pi r^2 + \frac{2}{3}\pi r^3$$

$$\frac{2}{3}r^3 + 60r^2 - 558 = 0$$

radius = $\boxed{3 \text{ cm}}$

$$\begin{array}{r|rrrr} 3 & \frac{2}{3} & 60 & 0 & -558 \\ & & 2 & 186 & 558 \end{array}$$

$$\frac{2}{3}r^3 + 62r^2 + 186r = 0$$

$$\frac{2}{3}r^2 + 62r + 186 = 0$$

∴ solutions are negative

$$6. \quad (2x+y)(2x-y) = 4x^2 - y^2$$

7.

$$\begin{array}{r} 6 - y^2 \\ 10 - y^2 \overline{) 60 - 16y^2 + y^4} \\ \underline{-60 + 6y^2} \\ -10y^2 + y^4 \\ \underline{+10y^2 + y^4} \\ 0 \end{array}$$

$$\boxed{6 - y^2}$$

$$8. \quad \begin{array}{r|rrrrr} -5 & 1 & 6 & 6 & 0 & 0 \\ & & -5 & -5 & -5 & 25 \\ \hline & 1 & 1 & 1 & -5 & 25 \end{array}$$

$$\boxed{x^3 + x^2 + x - 5 + \frac{25}{x+5}}$$

9. 5 real zeros (since there are 5 x-intercepts)

10. least possible degree: 5

of turning points: 4

of real zeros: 5

positive leading coefficient

as $x \rightarrow \infty$, $f(x) \rightarrow \infty$

as $x \rightarrow -\infty$, $f(x) \rightarrow -\infty$

> end behavior

11. $f(x) = 0$ when $x = -1, 1, 2$ ← multiplicity of 2
 multiplicity of 3
 multiplicity of 1

$$f(x) = -(x+1)^3(x-1)(x-2)^2$$

↑
 Make sure you have a negative leading coefficient!

12. $8y^3 - 4y^2 - 50y + 25$
 $4y^2(2y-1) - 25(2y-1)$
 $(4y^2-25)(2y-1)$
 $(2y+5)(2y-5)(2y-1)$

13. $24n^2 + 3n^5$
 $3n^2(8+n^3)$
 $3n^2(2+n)(4-2n+n^2)$

Know your rules for factoring sums/differences of cubes!

14. $2x^4 - 2x^3 - 8x^2 + 8x$
 $2x(x^3 - x^2 - 4x + 4)$
 $x^2(x-1) - 4(x-1)$
 $(x^2-4)(x-1)$
 $2x(x+2)(x-2)(x-1)$

15. $y^5 + 27y^2$
 $y^2(y^3 + 27)$
 $y^2(y+3)(y^2-3y+9)$

16. $3 \mid 2 \quad -3 \quad -8 \quad -3$
 $\quad \quad 6 \quad 9 \quad 3$

 $2 \quad 3 \quad 1 \quad 0$
 $2x^2 + 3x + 1$
 $(2x+1)(x+1)$

Given!
 $f(x) = (x-3)(2x+1)(x+1)$

17. $7 \mid 3 \quad -19 \quad -22 \quad 56$
 $\quad \quad 21 \quad 14 \quad -56$

 $3 \quad 2 \quad -8 \quad 0$
 $3x^2 + 2x - 8$

$f(x) = (x-7)(3x-4)(x+2)$

18. Possible rational zeros: $\pm 1, \pm 3, \pm 9$

19. Possible rational zeros: $\pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{8}{3}$

20. $g(x) = x^3 - x^2 - x + 1$ $\{x = 1, -1\}$
 multiplicity of 2

$$\begin{array}{r|rrrr} 1 & 1 & -1 & -1 & 1 \\ & & 1 & 0 & -1 \\ \hline & 1 & 0 & -1 & 0 \end{array}$$

$x^2 - 1 = 0$ $x = \pm 1$

21. $f(x) = 2x^3 - 5x^2 - 2x + 2$ $\{x = 1/2, 1 \pm \sqrt{3}\}$

$$\begin{array}{r|rrrr} 1/2 & 2 & -5 & -2 & 2 \\ & & 1 & -2 & -2 \\ \hline & 2 & -4 & -4 & 0 \\ & & 2x^2 - 4x - 4 = 0 \\ & & x^2 - 2x - 2 = 0 \end{array}$$

$x = \frac{2 \pm \sqrt{4 - 4(1)(-2)}}{2}$
 $x = \frac{2 \pm 2\sqrt{3}}{2}$

22. $f(x) = x^3 - 2x - 4$ $\{x = 2, -1 \pm i\}$

$$\begin{array}{r|rrrr} 2 & 1 & 0 & -2 & -4 \\ & & 2 & 4 & 4 \\ \hline & 1 & 2 & 2 & 0 \end{array}$$

$x^2 + 2x + 2 = 0$ $x = \frac{-2 \pm \sqrt{4 - 4(1)(2)}}{2}$
 $x = \frac{-2 \pm 2i}{2}$

23. $h(x) = 4x^4 + x^3 + 25x^2 + 7x - 21$ $x = \{-1, 3/4, \pm i\sqrt{7}\}$

$$\begin{array}{r|rrrrr} -1 & 4 & 1 & 25 & 7 & -21 \\ & & -4 & 3 & -28 & 21 \\ \hline 3/4 & 4 & -3 & 28 & -21 & 0 \\ & & 3 & 0 & 21 & \\ \hline & 4 & 0 & 28 & 0 & \end{array}$$

$4x^2 + 28 = 0$
 $x^2 = -7$
 $x = \pm i\sqrt{7}$

$$\begin{aligned}
 24. \quad f(x) &= (x+2)(x-i)(x+i)(x-\sqrt{3})(x+\sqrt{3}) \\
 &= (x+2)(x^2+1)(x^2-3) \\
 &= (x+2)(x^4-2x^2-3) \\
 f(x) &= x^5-2x^3-3x+2x^4-4x^2-6
 \end{aligned}$$

$$f(x) = x^5 + 2x^4 - 2x^3 - 4x^2 - 3x - 6$$

$$\begin{aligned}
 25. \quad f(x) &= x[x-(4-i)][x-(4+i)] \\
 f(x) &= x[(x-4)+i][(x-4)-i] \\
 f(x) &= x[(x-4)^2 - i^2] \\
 &= x(x^2 - 8x + 17) \\
 f(x) &= x^3 - 8x^2 + 17x
 \end{aligned}$$

$$\begin{aligned}
 26. \quad f(x) &= [x-(1+\sqrt{3})][x-(1-\sqrt{3})] \\
 f(x) &= [(x-1)-\sqrt{3}][(x-1)+\sqrt{3}] \\
 &= (x-1)^2 - 3 \\
 f(x) &= x^2 - 2x - 2
 \end{aligned}$$

$$\begin{aligned}
 27. \quad \text{L.C.} &= -6, \text{ degree} = 4, \text{ as } x \rightarrow \infty, r(x) \rightarrow -\infty \\
 &\text{as } x \rightarrow -\infty, r(x) \rightarrow -\infty
 \end{aligned}$$

$$\begin{aligned}
 28. \quad \text{L.C.} &= -16, \text{ degree} = 3, \text{ as } x \rightarrow \infty, q(x) \rightarrow -\infty \\
 &\text{as } x \rightarrow -\infty, q(x) \rightarrow \infty
 \end{aligned}$$

$$\begin{aligned}
 29. \quad \text{max: } &5.092, 13.494 \\
 \text{min: } &4.414
 \end{aligned}$$

$$\begin{aligned}
 30. \quad \text{max: } &0 \\
 \text{min: } &-8.870
 \end{aligned}$$

$$f(x) = -2x^4 + 7x^2 - 4$$

31. a) $g(x) = f(\frac{1}{4}x)$ horizontal stretch by 4
 $= -2(\frac{1}{4}x)^4 + 7(\frac{1}{4}x)^2 - 4$

$$\boxed{g(x) = -\frac{1}{128}x^4 + \frac{7}{16}x^2 - 4}$$

b) $g(x) = 4f(x)$ vertical stretch by 4
 $= 4(-2x^4 + 7x^2 - 4)$

$$\boxed{g(x) = -8x^4 + 28x^2 - 16}$$

32. a) $g(x) = f(x-5)$ horizontal shift right 5
 $\boxed{g(x) = -2(x-5)^4 + 7(x-5)^2 - 4}$

b) $g(x) = 0.25f(x)$ vertical compression by $\frac{1}{4}$
 $\boxed{g(x) = -\frac{1}{2}x^4 + \frac{7}{4}x^2 - 1}$

33. a. $g(x) = -4(x+2)^3 + 5$

b. $g(x) = f(5x) + 1$
 $= 4(5x)^3 - 5 + 1$

$$\boxed{g(x) = 500x^3 - 4}$$

c. $g(x) = 3f(x-3)$
 $= 3(4(x-3)^3 - 5)$

$$\boxed{g(x) = 12(x-3)^3 - 15}$$

d. $g(x) = f(-x)$
 $\boxed{g(x) = -4x^3 - 5}$

graphs on separate page

$f(x) = 4x^3 - 5$