Example #4

The height h in feet of a baseball t seconds after it is hit can be modeled by the function $h(t) = -16t^2 + 96t + 3$.

a. Find the maximum height of the baseball.

VERTEX $\quad -\frac{b}{2a} = \frac{-96}{2(-16)}$

maximum height occurs after 3 seconds

maximum height = 147 feet

$h(3) = -16(3^2) + 96(3) + 3$

b. How long does the ball take to hit the ground?

$-16t^2 + 96t + 3 = 0$

$a = -16$

$b = 96$

$c = 3$

$t = \frac{-96 \pm \sqrt{96^2 - 4(-16)(3)}}{-32}$

$t = \frac{-96 \pm \sqrt{9408}}{-32} \approx$ -0.031 seconds, 6.03 seconds

Additional Practice

1. The quadratic function that approximates the height of a javelin thrown is $h(t) = -0.08t^2 + 4.48$, where t is the time in seconds after it is thrown and h is the javelin's height in feet. How long will it take for the javelin to hit the ground?

$-0.08t^2 + 4.48 = 0$

$-0.08t^2 = -4.48$

$t^2 = 56$

$t \approx 7.48$ seconds

2. a. Marilyn hit a golf ball on the ground with her driver. Use the general function for a projectile to write a function that shows the height in feet of her golf ball as a function of time. The ball was hit with an initial vertical velocity of 100 feet per second.

$h(t) = -16t^2 + 100t$

b. How long will Marilyn's golf ball stay in the air?

$-16t^2 + 100t = 0$

$-16t\left(t - \frac{25}{4}\right) = 0$

$t = 6.25$ seconds

3. After the semester is over, you discover that the math department has changed textbooks so the bookstore won't buy back your nearly new book. You and your friend Frank decide to get creative. You go to the roof of a twelve-story building and look over the edge to the reflecting pool 160 feet below. You drop your book over the edge at the same instant that Frank chucks his book straight ***down*** (so the initial vertical velocity is negative) at 48 feet per second.

a. Write a function h in standard form for your book's height in feet in terms of the time t in seconds after the book is dropped.

$h(t) = -16t^2 + 160$ No V_0

b. Write a function f in standard form for the height of Frank's book in terms of the time t in seconds after the book is "chucked".

$f(t) = -16t^2 - 48t + 160$

c. By how many seconds does his book beat yours into the water? Show your work to justify your answer.

$-16t^2 + 160 = 0$

$-16t^2 = -160$

$t^2 = 10$

$t \approx 3.16$ seconds

ME

$-16t^2 - 48t + 160 = 0$

$t^2 + 3t - 10 = 0$

$(t+5)(t-2) = 0$

$t = -5, 2$

FRANK $t = 2$ seconds

Frank's book beats mine by 1.16 seconds

4. The height $h(t)$ measured in feet of an object dropped by an astronaut on the moon can be approximated by $h(t) = h_0 - 2.7t^2$, where h_0 is the height from which the object was dropped. About how long would it take an object to fall to the surface of the moon ($h = 0$) if it were dropped by an astronaut from a height of 6 feet?

$6 - 2.7t^2 = 0$

$-2.7t^2 = -6$

$t^2 = 2.\overline{2}$

$t \approx 1.49$ seconds

5. The height h in feet of a baseball t seconds after it is hit can be modeled by the function $h(t) = -16t^2 + 80t + 2$.

a. Find the maximum height of the baseball. $\frac{-80}{2(-16)} = 2.5$ $h(2.5) =$ 102 feet

b. How long does the ball take to hit the ground?

$-16t^2 + 80t + 2 = 0$

$t = \frac{-80 \pm \sqrt{80^2 - 4(-16)(2)}}{2(-16)}$

$t = \frac{-80 \pm \sqrt{6272}}{-32} = 0.025$ seconds, 4.975 seconds

6. A juggler tosses a ball into the air. The ball leaves the juggler's hand 4 feet above the ground and has an initial vertical velocity of 30 feet per second. The juggler catches the ball when it falls back to a height of 3 feet. How long is the ball in the air?

$-16t^2+30t+4=3$

$-16t^2+30t+1=0$

$h(t)=-16t^2+30t+4$

$t=\frac{-30\pm\sqrt{30^2-4(-16)(1)}}{2(-16)}=\frac{-30\pm\sqrt{964}}{-32}\approx$ 1.91 seconds, -0.033

a. What if the ball leaves the juggler's hand with an initial vertical velocity of 40 feet per second? How long is the ball in the air?

$-16t^2+40t+4=3$

$-16t^2+40t+1=0$

$t=\frac{-40\pm\sqrt{40^2-4(-16)(1)}}{-32}$

$t=\frac{-40\pm\sqrt{1664}}{-32}\approx$ 2.525 seconds, -0.025

7. A lacrosse player throws a ball in the air from an initial height of 7 feet. The ball has an initial vertical velocity of 90 feet per second. Another player catches the ball when it is 3 feet above the ground. How long is the ball in the air?

$h(t)=-16t^2+90t+7$

$-16t^2+90t+7=3$

$-16t^2+90t+4=0$

$t=\frac{-90\pm\sqrt{90^2-4(-16)(4)}}{-32}=\frac{-90\pm\sqrt{8356}}{-32}$

-0.044, 5.67 seconds