

LESSON
4-4
Practice C
Properties of Logarithms

Express as a single logarithm. Simplify, if possible.

1. $\log_6 12 + \log_6 18$

2. $\log_3 81 - \log_3 27$

3. $\log_4 128 - \log_4 8$

4. $\log_6 18 + \log_6 72$

5. $\log_5 3125 - \log_5 25$

6. $\log_8 128 + \log_8 256$

7. $\log_5 5 + \log_5 125$

8. $\log_2 256 - \log_2 64$

9. $\log_3 8019 - \log_3 99$

10. $\log_8 80 + \log_8 51.2$

11. $\log_7 13.3 - \log_7 1.9$

12. $\log_{10} 125 + \log_{10} 80$

Evaluate. Round to the nearest hundredth.

13. $\log_8 8^6$

14. $2^{\log_2 8^x}$

15. $\log_2 16^5$

16. $\log_3 3^{(2x+1)}$

17. $\log_4 16^{(x-1)}$

18. $5^{\log_5 17}$

19. $\log_3 5^2$

20. $\log_5 \left(\frac{1}{125} \right)^2$

21. $\log_6 \left(\frac{1}{6^4} \right)^3$

22. $\log_4 20^2$

23. $\log_9 27^4$

24. $\log_2 10$

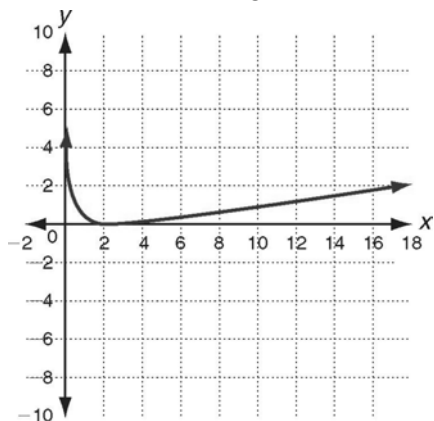
Solve.

25. Carmen has a painting presently valued at \$5000. An art dealer told her the painting would appreciate at a rate of 6% per year. In how many years will the painting be worth \$8,000?

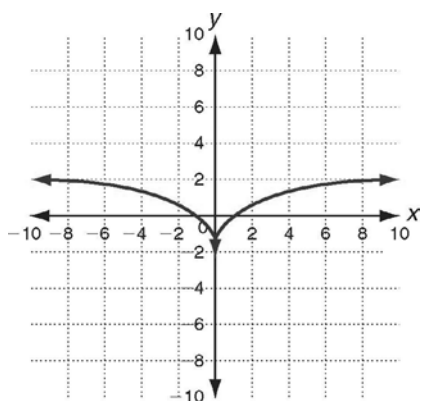
a. Write a logarithmic expression.

b. Simplify your expression.

4. Domain: $x > 0$; range: $y > 0$



5. Domain: $x \neq 0$; range: all real numbers



4-4 PROPERTIES OF LOGARITHMS

Practice A

- | | |
|---------------------|---------------------------|
| 1. 4 | 2. 64; 64; 6 |
| 3. 3125; 3125; 5 | 4. $\log_{10} 10,000 = 4$ |
| 5. $\log_6 6 = 1$ | 6. $\log_8 64 = 2$ |
| 7. $\log_5 25 = 2$ | 8. $\log_3 3 = 1$ |
| 9. $\log_2 32 = 5$ | 10. $\log_4 16 = 2$ |
| 11. $\log_6 36 = 2$ | 12. $\log_5 125 = 3$ |
| 13. 4 | 14. 4 |
| 15. 9 | 16. 4 |
| 17. 12 | 18. 2 |
| 19. 1.59 | 20. 1.77 |
| 21. 1.46 | 22. 10^{22} ergs |

Practice B

- | | |
|---------------------------|----------------------|
| 1. $\log_3 243 = 5$ | 2. $\log_2 128 = 7$ |
| 3. $\log_{10} 10,000 = 4$ | 4. $\log_6 216 = 3$ |
| 5. $\log_3 81 = 4$ | 6. $\log_4 4096 = 6$ |

Problem Solving

- $\text{pH} = -\log(0.0000629)$
 - $\text{pH} = 4.2$
- $\text{pH} = -\log(0.0000032)$
 - $\text{pH} = 5.5$
 - Rainwater in eastern Ohio has a lower pH than that in central California by 1.3 units.
- D
- J
- A
- H

Reading Strategies

- $4^2 = 16$; $\log_5 0.2 = -1$; $\log_6 1 = 0$
- $\log_b 1 = 0$ is the same as $b^0 = 1$ and any number to the 0 power is 1.
- $f^{-1}(x) = \log_4 x$
- $g^{-1}(x) = \log_{\frac{1}{2}} x$
 - Domain of $g(x)$ is all real numbers
range of $g(x)$ is $y > 0$
domain of $g^{-1}(x)$ is $x > 0$
range of $g^{-1}(x)$ is all real numbers.

- | | |
|----------------------|-----------------------------|
| 7. $\log_2 8 = 3$ | 8. $\log_{10} 100 = 2$ |
| 9. $\log_4 64 = 3$ | 10. $\log_2 64 = 6$ |
| 11. $\log_3 243 = 5$ | 12. $\log_6 36 = 2$ |
| 13. 6 | 14. $x - 5$ |
| 15. 30 | 16. 1 |
| 17. 5 | 18. 8 |
| 19. 0 | 20. 3.10 |
| 21. 1.43 ergs | 22. $1.26 \times 10^{18.1}$ |

Practice C

- | | |
|---------------------|----------------------------|
| 1. $\log_6 216 = 3$ | 2. $\log_3 3 = 1$ |
| 3. $\log_4 16 = 2$ | 4. $\log_6 1296 = 4$ |
| 5. $\log_5 125 = 3$ | 6. $\log_8 32,768 = 5$ |
| 7. $\log_5 625 = 4$ | 8. $\log_2 4 = 2$ |
| 9. $\log_3 81 = 4$ | 10. $\log_8 4096 = 4$ |
| 11. $\log_7 7 = 1$ | 12. $\log_{10} 10,000 = 4$ |
| 13. 6 | 14. 8^x |
| 15. 20 | 16. $2x + 1$ |
| 17. $2x - 2$ | 18. 17 |

19. 2.93
 21. -12
 23. 6
 25. a. $\log_{1.06} 1.6$
 b. 8 years

20. -6
 22. 4.32
 24. 3.32

Reteach

1. 3
 3. $\log_9 (3 \cdot 27)$; $\log_9 81$; 2
 4. $2 \cdot 3 = 6$
 6. $3 \log_9 81$; $3 \cdot 2 = 6$
 8. 75
 2. $\log_2 16$; 4
 5. $4 \cdot 4 = 16$
 7. $5y$
 9. $3x$

Challenge

1. Both expressions equal $\frac{3}{2}$.
 2. Result is $\frac{3}{2}$; formula is easier to compute.
 3. Result is $\frac{6}{5}$; formula is easier to compute.
 4. $\log_a b \cdot \log_b c = \log_a c$

$$\log_a b \cdot \log_b c = \frac{\log b}{\log a} \cdot \frac{\log c}{\log b}$$

$$= \frac{\log c}{\log a} \cdot \frac{\log b}{\log b} = \frac{\log c}{\log a} = \log_a c$$

 5. $\log_2 13 = \frac{\log 13}{\log 2} \approx 3.7$
 6. $\log_2 32 = 5$; possible answer: using the Chain Rule is much easier.

Problem Solving

1. a. $7.8 = \frac{2}{3} \log \left(\frac{E}{10^{11.8}} \right)$
 b. $23.5 = \log E$
 c. Yes; by the definition of logarithm; $E = 10^{23.5}$
 d. They are both correct; $10^{23.5} = 3.16 \times 10^{23}$.
 2. A
 4. C
 3. G
 5. F

Reading Strategies

- True; Product Property
- True; Quotient Property
- False; Power Property
- False: Inverse Property
- $5 \log x$; Power Property
- $\log x$; Quotient Property
- 1; Product Property
- $x \log x$; Power Property
- $2 \log xy$; Quotient Property and Power Property
- 7; Product Property and Inverse Property
- $x = 1$

4-5 EXPONENTIAL AND LOGARITHMIC EQUATIONS AND INEQUALITIES

Practice A

- | | |
|---------------------------|-----------------------------|
| 1. 129; $x \approx 0.645$ | 2. $x \approx 3.43$ |
| 3. $x \approx 0.161$ | 4. $x = -3$ |
| 5. $x = 2$ | 6. $x \approx 1.686$ |
| 7. $x = 9$ | 8. $x = -10$ |
| 9. $x = -2$ | 10. $x = 8$ |
| 11. $x = 3125$ | 12. $x = 10$ (or $x = -2$) |
| 13. $x = 9000$ | 14. $x = 2.5$ |
| 15. $x = 2$ | 16. 19 years |

Practice B

- | | |
|-----------------------|-----------------------|
| 1. $x \approx 0.9307$ | 2. $x \approx 4.5449$ |
| 3. $x = -4$ | 4. $x = 3$ |
| 5. $x \approx -6.67$ | 6. $x = 6$ |
| 7. $x = -8$ | 8. $x = -0.6$ |
| 9. $x = 5$ | 10. $x = 256$ |
| 11. $x = 9$ | 12. $x = 22$ |
| 13. $x = 10^{15}$ | 14. $x = 20$ |
| 15. $x = 16$ | 16. $x = 56$ |
| 17. $x = \pm 2$ | 18. $x = -1, -2$ |
| 19. $x < 11$ | 20. $x = 10,000$ |
| 21. $x = 4$ | 22. 2009 |