

15.
$$\frac{1}{\cot^2 x + 1} = \frac{1}{\csc^2 x} = \sin^2 x$$

16.
$$\frac{\sec^2 x - 1}{\sec x - 1} = \frac{(\sec x - 1)(\sec x + 1)}{\sec x - 1} = \sec x + 1$$

17.
$$\frac{\sin^2 \alpha - \cos^2 \alpha}{\sin^2 \alpha - \sin \alpha \cos \alpha} = \frac{(\sin \alpha + \cos \alpha)(\sin \alpha - \cos \alpha)}{\sin \alpha(\sin \alpha - \cos \alpha)} = \frac{\sin \alpha + \cos \alpha}{\sin \alpha} = 1 + \cot \alpha$$

18.
$$\frac{\sin^3 \beta + \cos^3 \beta}{\sin \beta + \cos \beta} = \frac{(\sin \beta + \cos \beta)(\sin^2 \beta - \sin \beta \cos \beta + \cos^2 \beta)}{\sin \beta + \cos \beta} = 1 - \sin \beta \cos \beta$$

19.
$$\tan^2 \theta(\csc^2 \theta - 1) = \tan^2 \theta(\cot^2 \theta)$$

20.
$$\csc^2 x(1 - \cos^2 x) = \csc^2 x(\sin^2 x) = 1$$

$$= \tan^2 \theta \left(\frac{1}{\tan^2 \theta} \right) = 1$$

21.
$$\begin{aligned} \tan\left(\frac{\pi}{2} - x\right) \sec x &= \cot x \sec x \\ &= \frac{\cos x}{\sin x} \cdot \frac{1}{\cos x} = \frac{1}{\sin x} = \csc x \end{aligned}$$

22.
$$\frac{\sin(-x) \cot x}{\sin\left(\frac{\pi}{2} - x\right)} = \frac{(-\sin x)\left(\frac{\cos x}{\sin x}\right)}{\cos x} = -1$$

For #25 – 36, answers will vary