Solving Three Variable Systems

- Solutions will be ordered triples (x, y, z).
- Possible solutions are: an ordered triple, infinitely many solutions or no solutions.
- Can use substitution or elimination to solve three variable systems.

$(1, 4, -4)$

Let's try one together.

(1) Solve: $x - 4y + 3z = -27$
$2x + 2y - 3z = 22$
$4z = -16$ $\longrightarrow z = -4$

$x - 4y + 3(-4) = -27$ $\quad x - 4y - 12 = -27$ $\quad x - 4y = -15$

$2x + 2y - 3(-4) = 22$ $\quad 2x + 2y + 12 = 22$ $\quad 2(2x + 2y = 10)$

$$\begin{array}{r} 1 - 4y = -15 \\ -1 \qquad\quad -1 \\ \hline -4y = -16 \\ \overline{-4} \quad \overline{-4} \end{array}$$

$y = 4$

$$\begin{array}{r} x - \cancel{4y} = -15 \\ + \quad 4x + \cancel{4y} = 20 \\ \hline 5x = 5 \end{array}$$

$x = 1$

(2) $2x - y + 2z = 15$ $\cdot -1$
$+ \; -x + y + z = 3$
$3x - y + 2z = 18$

$(x + 3z = 18)$

$2x + 3z = 21$

$$\begin{aligned} -x - 3z &= -18 \\ +\; 2x + 3z &= 21 \\ \hline x &= 3 \end{aligned}$$

$3 + 3z = 18$
$3z = 15$
$z = 5$

$3 + y + 5 = 3$
$y + 2 = 3$
$y = 1$

$\boxed{(3, 1, 5)}$

(3) $2x - 5y + z = 5$

$2(3x + 2y - z = 17)$

$4x - 3y + 2z = 17$

$6x + 4y - 2z = 34$

$2(5) - 5(1) + z = 5$

$10 - 5 + z = 5$

$z = 0$

$\boxed{(5, 1, 0)}$

$-2(5x - 3y = 22)$

$10x + y = 51$

$-10x + 6y = -44$

$10x + y = 51$

$7y = 7$

$y = 1$

$10x + 1 = 51$

$x = 5$

(4) $3x - 2y + 4z = 15$
$x - y + z = 3$
$x + 4y - 5z = 0$

(3, 3, 3)

(5) $2x + 3y + 4z = 2$
$5x - 2y + 3z = 0$
$x - 5y - 2z = -4$

~~(6)~~ 5) $a + b = 3$

$-b + c = 3$

$a + 2c = 10$

$a + b = 3$

$c - b = 3$

$-1(a + c = 6) \rightarrow$

$a + 2c = 10$

$-a - c = -6$

$c = 4$

$a = 2$

$b = 1$

$\boxed{(2, 1, 4)}$