Recall: The domain of linear and quadratic functions is all real numbers. So, what happens when you solve an inequality that is based on a function for which the domain is *NOT* all real numbers? So far in this course, we have studied two functions that fit this description. The domain of a rational function has at least one value excluded from its domain, and the domain of a square root function has a restriction. So, when solving a rational or a square root inequality, you must account for the fact that the function's domain is *NOT* all real numbers.

Notes: 5.8 Solving Radical Inequalities

Solve each inequality. Answers should be given using interval notation. Use a number line to assist you.

1. $\sqrt{x+5}-1\leq 4$

① Identify restrictions on domain

$x+5\geq 0$

$x\geq -5$

$x\geq -5$

$\sqrt{x+5}-1=4$

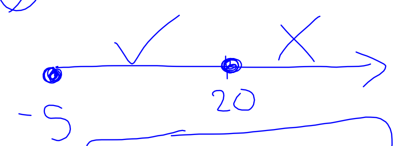
$\sqrt{x+5}=5$

$x+5=25$

$x=20$

② Solve related equation

③ Number Line



$\boxed{[-5, 20]}$

2. $\sqrt{3x+3} \leq 6$

① restrictions

$3x+3 \geq 0$

$x \geq -1$

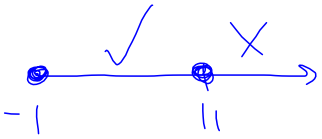
② Solve

$\sqrt{3x+3} = 6$

$3x+3 = 36$

$3x = 33$

$x = 11$



Solution $[-1, 11]$

3. $\sqrt{8x+1} \geq 7$

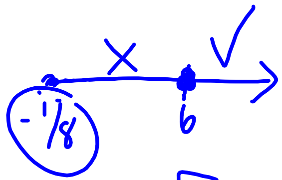
$$\boxed{\begin{aligned} 8x+1 &\geq 0 \\ x &\geq -1/8 \end{aligned}}$$

$\sqrt{8x+1} = 7$

$8x+1 = 49$

$8x = 48$

$x = 6$



$[6, \infty)$

4.

$$\sqrt[3]{x+3} \geq 2$$

No restrictions

$[5, \infty)$

$\sqrt[3]{x+3} \leq 2$

$(-\infty, 5]$

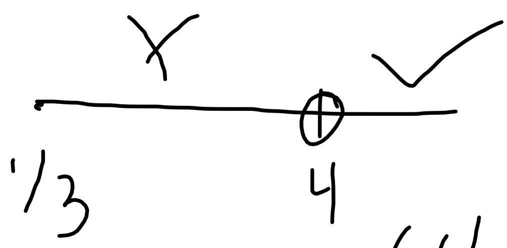
5.

$3x - 1 \geq 0$ $x + 7 \geq 0$

$$\sqrt{3x-1} > \sqrt{x+7}$$

$x \geq 1/3$

$x \geq -7$



$(4, \infty)$