1-1 Exploring Transformations

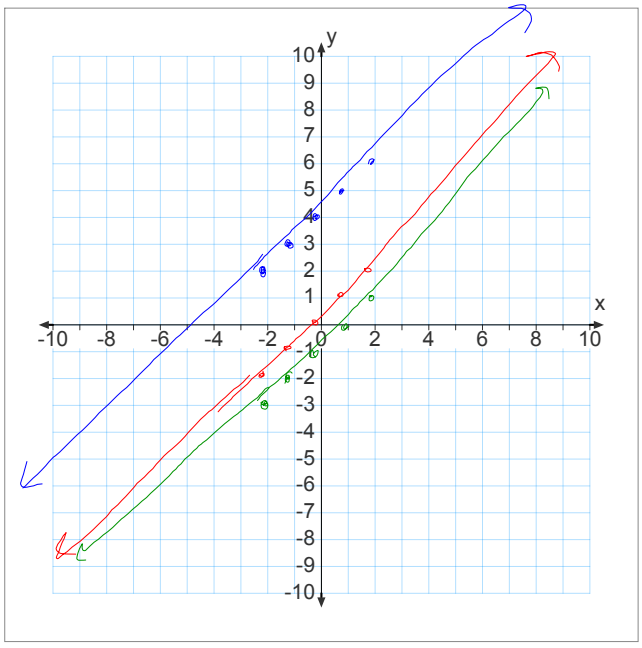
$$f(x) = a(b(x - h)) + k$$

Translations

- The shape and size of the parent graph stays the same.
- The entire graph just changes position.
- Translations are also known as "shifts" and can occur horizontally or vertically.

K

$f(x) = x$		$g(x) = f(x) + 4$		$h(x) = f(x) - 1$	
		$g(x) = x + 4$		$h(x) = x - 1$	
x	$f(x)$	x	$g(x)$	x	$h(x)$
-2	-2	-2	2	-2	-3
-1	-1	-1	3	-1	-2
0	0	0	4	0	-1
1	1	1	5	1	0
2	2	2	6	2	1



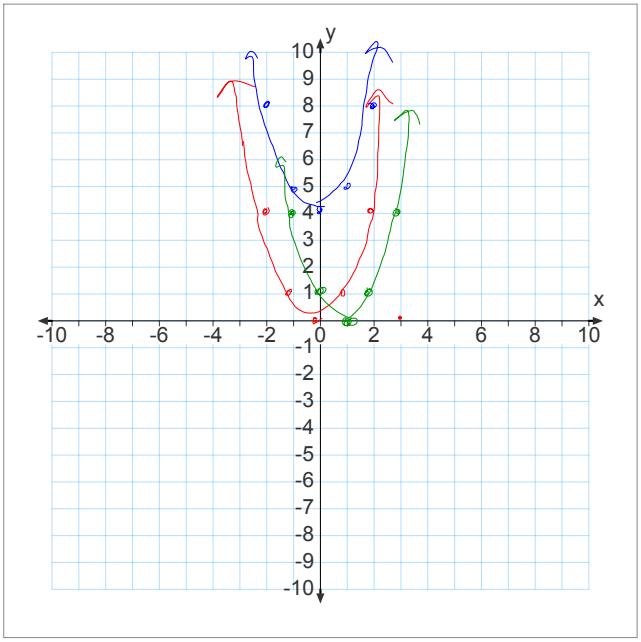
k h

$f(x)=x^2$	$g(x)=f(x)+4$	$h(x)=f(x-1)$
	$g(x)=x^2+4$	$h(x)=(x-1)^2$

x	$f(x)$
-2	4
-1	1
0	0
1	2
2	4

x	$g(x)$
-2	8
-1	5
0	4
1	5
2	8

x	$h(x)$
-1	4
0	1
1	0
2	1
3	4



Reflections

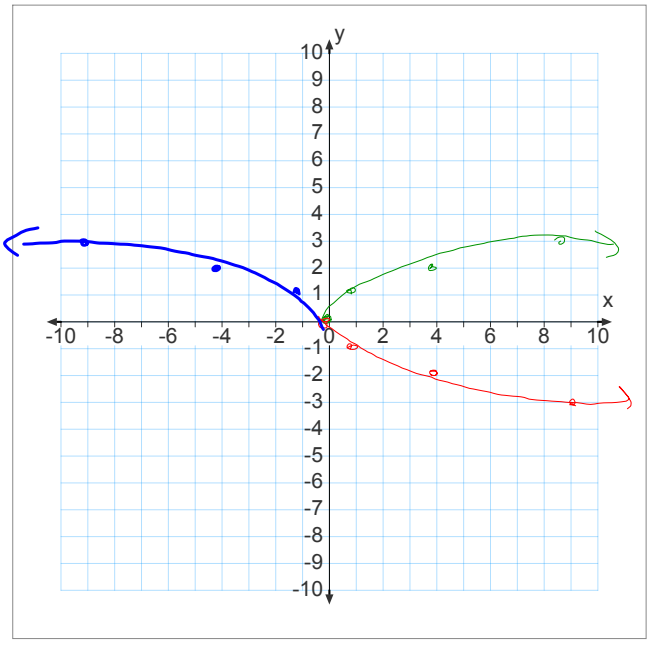
- The shape and size of the parent graph stays the same.
- A reflection is the mirror image of a graph over the x- or y- axis.

$f(x)=\sqrt{x}$	$g(x)=-f(x)$	$h(x)=f(-x)$
	$g(x)=-\sqrt{x}$	$h(x)=\sqrt{-x}$

x	$f(x)$
0	0
1	1
4	2
9	3

x	$g(x)$
0	0
1	-1
4	-2
9	-3

x	$h(x)$
0	0
-1	1
-4	2
-9	3



Stretches and Compressions

- Cause a distortion in the shape of the parent graph.
- ***Stretch*** pulls the graph away from the axes.
- ***Compression*** pushes the graph towards the axes.

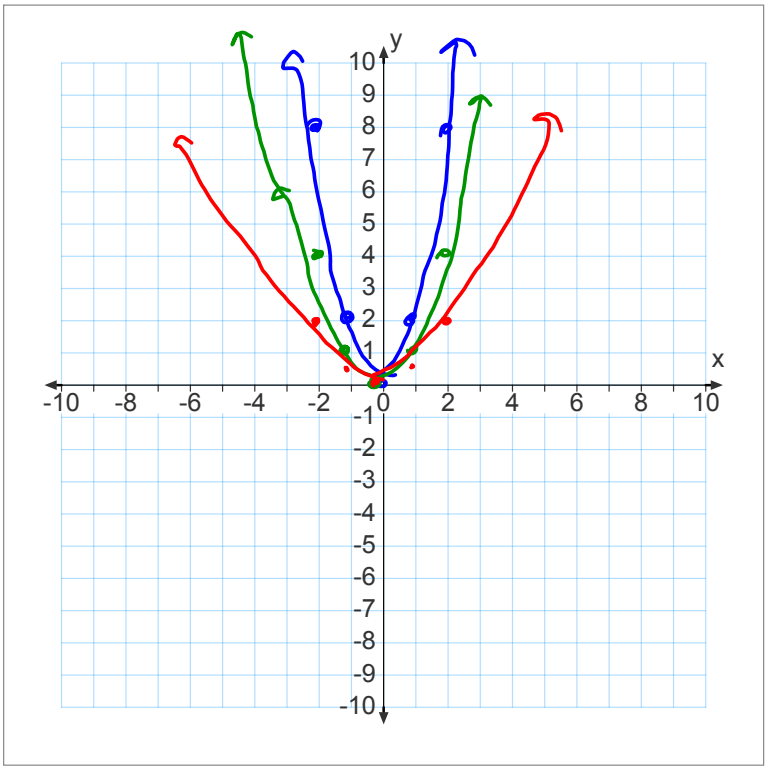
a

$f(x)=x^2$	$g(x)=2f(x)$	$h(x)=\frac{1}{2}f(x)$
	$g(x)=2x^2$	$h(x)=\frac{1}{2}x^2$

x	$f(x)$
−2	4
−1	1
0	0
1	1
2	4

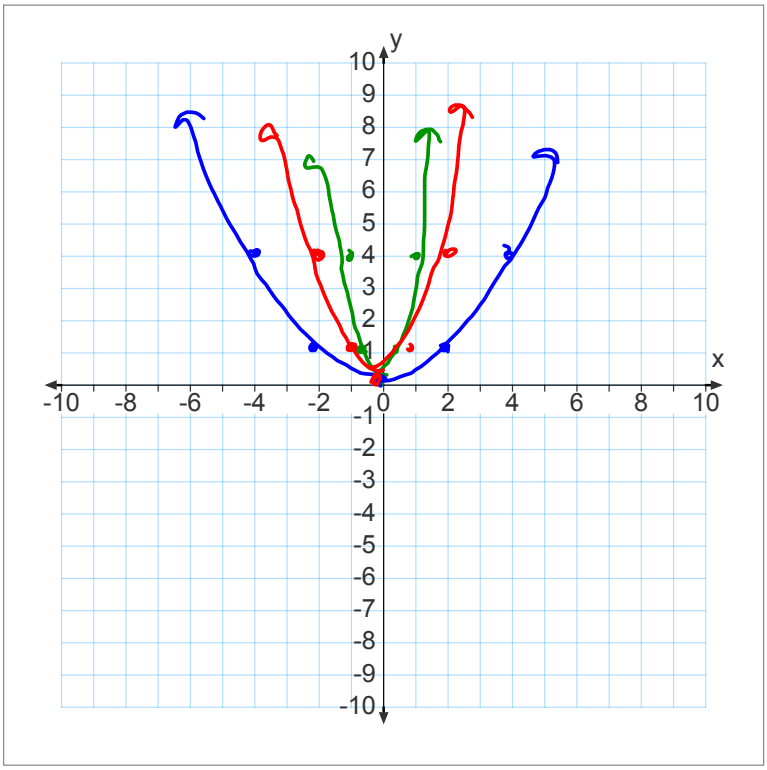
x	$g(x)$
−2	8
−1	2
0	0
1	2
2	8

x	$h(x)$
-2	2
-1	½
0	0
1	½
2	2



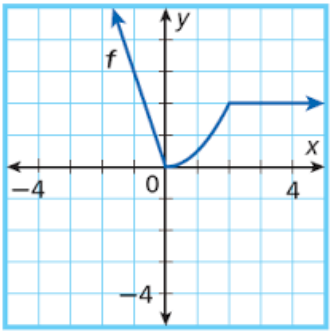
b

$f(x)=x^2$		$g(x)=f(2x)$		$h(x)=f\left(\frac{1}{2}x\right)$	
		$g(x)=(2x)^2$		$h(x)=\left(\frac{1}{2}x\right)^2$	
x	$f(x)$	x	$g(x)$	x	$h(x)$
-2	4	-1	4	-4	4
-1	1	$-\frac{1}{2}$	1	-2	1
0	0	0	0	0	0
1	1	$\frac{1}{2}$	1	2	1
2	4	1	4	4	4



INPUT		OUTPUT	
f(x - h)	• horizontal translation • shifts graph h units $h>0 \rightarrow$ right $h<0 \rightarrow$ left $(x,y) \rightarrow (x+h, y)$	f(x) + k	• vertical translation • shifts graph k units $k>0 \rightarrow$ up $k<0 \rightarrow$ down $(x,y) \rightarrow (x, y+k)$
f(-x)	reflection in y-axis $(x,y) \rightarrow (-x,y)$	-f(x)	reflection in x-axis $(x,y) \rightarrow (x,-y)$
f(bx)	$\lvert b\rvert>1 \rightarrow$ horizontal compression $\lvert b\rvert<1$ horizontal stretch $(x,y) \rightarrow (\frac{1}{b}x, y)$ • horizontal stretch or compression by a factor of $\frac{1}{b}$	af(x)	$\lvert a\rvert>1 \rightarrow$ vertical stretch $\lvert a\rvert<1 \rightarrow$ vertical compression $(x,y) \rightarrow (x, ay)$ • vertical stretch/comp by a factor of a

Use a table to perform each transformation of $y=f(x)$.



(A) horizontal compression by a factor of 0.5

(B) translation 3 units up

(C) reflection across the *x*-axis