9.2 Ellipses

Definition of an Ellipse

An **ellipse** is the set of all points, P, in a plane the sum of whose distances from two fixed points, F_1 and F_2, is constant (see **Figure 9.3**). These two fixed points are called the **foci** (plural of **focus**). The midpoint of the segment connecting the foci is the **center** of the ellipse.

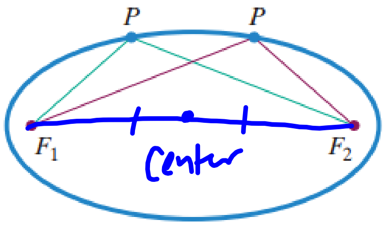
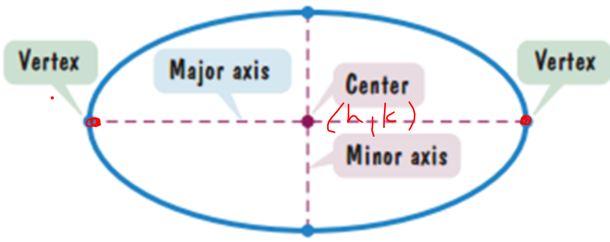


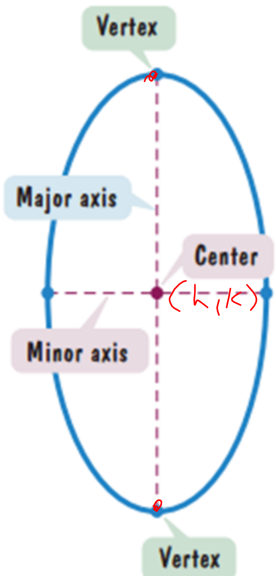
FIGURE 9.3

$a > b$



Major axis is horizontal

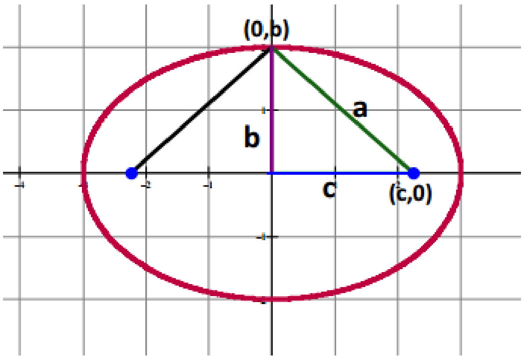
$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$



Major axis is vertical

$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$

Relationship between a, b, and c.



length of
Major Axis = 2a

length of
Minor Axis = 2b

Foci: $c^2 = a^2 - b^2$

c units from center

Find the indicated values and graph:

1.) $\frac{9x^2 + 4y^2}{36} = \frac{36}{36}$

$\frac{x^2}{4} + \frac{y^2}{9} = 1$

horizontal movement

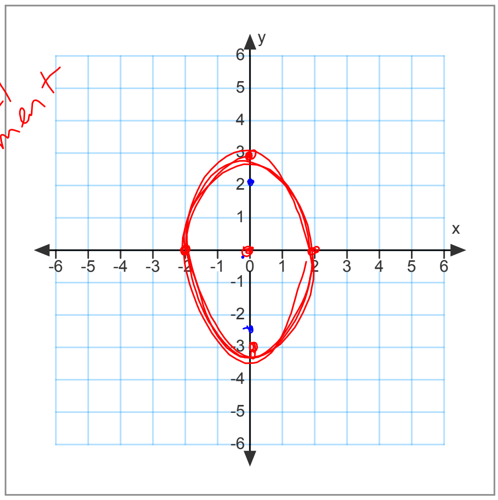
vertical movement

center: (0, 0)

vertices: (0, 3)
(0, -3)

co-vertices: (-2, 0)
(2, 0)

foci: $(0, \sqrt{5})$, $(0, -\sqrt{5})$



$c^2 = a^2 - b^2$

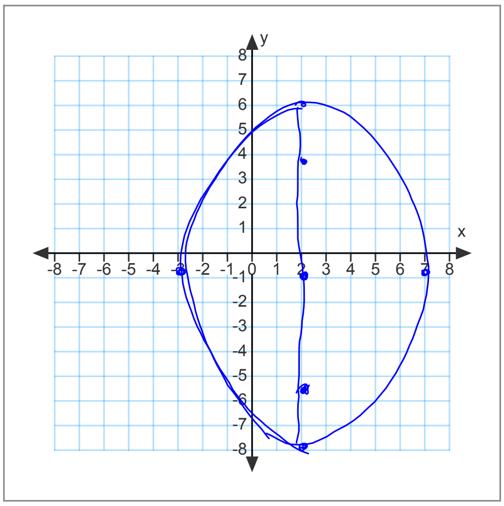
$c^2 = 9 - 4 = 5$ $c = \sqrt{5}$

Find the indicated values and graph:

2.) $49(x-2)^2+25(y+1)^2=1225$

$$\frac{49(x-2)^2+25(y+1)^2}{1225}=\frac{1225}{1225}$$

$$\frac{(x-2)^2}{25}+\frac{(y+1)^2}{49}=1$$



center: (2, -1)

vertices: endpoints of major axis
(2, 6), (2, -8)

co-vertices: endpoints of minor axis
(-3, -1), (7, -1)

foci: $\boxed{(2, -1+2\sqrt{6})(2, -1-2\sqrt{6})}$

$c^2=a^2-b^2=49-25=24$ $\quad c=\sqrt{24}=2\sqrt{6}$

Find the indicated values and graph:

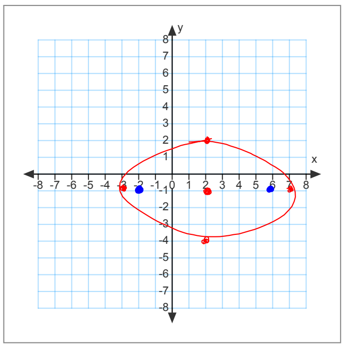
3.) $9x^2+25y^2-36x+50y-164=0$

$9x^2-36x+25y^2+50y=164$

$9(x^2-4x+4)+25(y^2+2y+1)=164$ $+36$ $+25$

$$\frac{9(x-2)^2+25(y+1)^2}{225}=\frac{225}{225}$$

$$\frac{(x-2)^2}{25}+\frac{(y+1)^2}{9}=1$$



center: (2, -1)

vertices:
(-3, -1), (7, -1)

co-vertices: (2, 2)
(2, -4)

foci: (6, -1), (-2, -1)

$c^2=25-9=16$

$c=4$