6.6 Functions and Their Inverses

*Functions *f* and *g* are inverses of each other iff $f(g(x)) = g(f(x)) = x$.

*Notation: The inverse of $f(x)$ is written $f^{-1}(x)$. This is read as "f inverse of x."

Examples

(1) Verify that $f(x) = 3x + 6$ and $f^{-1}(x) = \frac{1}{3}x - 2$ are inverses.

$f(f^{-1}(x)) = 3\left(\frac{1}{3}x - 2\right) + 6 = x - 6 + 6 = x$

$f^{-1}(f(x)) = \frac{1}{3}(3x+6) - 2 = x + 2 - 2 = x$

$f(f^{-1}(x)) = f^{-1}(f(x)) = x$ $\therefore$ they are inverse functions

(2) Verify that $f(x) = \sqrt{5x-2}$ and $f^{-1}(x) = \frac{x^2+2}{5}$, $x \geq 0$ are inverses.

$f(f^{-1}(x)) = \sqrt{\cancel{5}\left(\frac{x^2+2}{\cancel{5}}\right) - 2} = \sqrt{x^2+2-2} = \sqrt{x^2} = x$

$f^{-1}(f(x)) = \frac{(\sqrt{5x-2})^2 + 2}{5} = \frac{5x-2+2}{5} = \frac{5x}{5} = x$

To find the inverse of a relation or function, interchange *x* and *y*. Then solve for *y*. **Remember - think about what "inverse" means!**

Examples. Find the inverse of each function.

(3) $f(x) = 3x - 4$

$y = 3x - 4$

$x = 3y - 4$

$x + 4 = 3y$

$y = \frac{x+4}{3}$

$f^{-1}(x) = \frac{x+4}{3}$

(4) $f(x) = \frac{3x-2}{5}$

$x = \frac{3y-2}{5}$

$5x = 3y - 2$

$5x + 2 = 3y$

$\frac{5x+2}{3} = y$

$f^{-1}(x) = \frac{5x+2}{3}$

(5) Graph $f(x) = -\frac{1}{2}x - 5$. Then write the inverse and graph.

$x = -\frac{1}{2}y - 5$

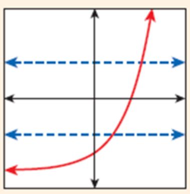
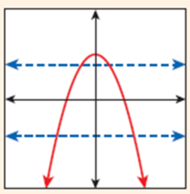
$x + 5 = -\frac{1}{2}y$

$y = -2x - 10 \leftarrow f^{-1}(x) = -2x - 10$

***Note: The graph of $f^{-1}(x)$ is a reflection of the graph of $f(x)$ over the line $y = x$.**

Recall that the vertical-line test can help you determine whether a relation is a function. Similarly, the *horizontal line* test can help you determine whether the inverse of a function is a function.

Horizontal-line Test

WORDS	EXAMPLES	
If any horizontal line passes through more than one point on the graph of a relation, the inverse relation is not a function.	 Inverse is a function.	 Inverse is not a function.

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Recall that to write the rule for the inverse of a function, you can ~~exchange~~ *x* and *y* and solve the equation for *y*. Because the values of *x* and *y* are switched, the domain of the function will be the range of its inverse and vice versa.

Examples. Find the inverse of each function. Determine whether the inverse is a function. State its domain and range.

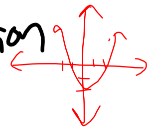
(6) $f(x) = x^2 - 2$

$x = y^2 - 2$

$\pm\sqrt{x+2} = \sqrt{y^2}$

$\boxed{y = \pm\sqrt{x+2}}$

Not a function



(7) $f(x) = (x + 3)^2, x \geq -3$

$\sqrt{x} = \sqrt{(y+3)^2}$

$\sqrt{x} = y + 3$

$\boxed{f^{-1}(x) = -3 + \sqrt{x}}$

function

	f(x)	Inverse
(6)	D: $(-\infty, \infty)$	D: $[-2, \infty)$
	R: $[-2, \infty)$	R: $(-\infty, \infty)$
(7)	D: $[-3, \infty)$	D: $[0, \infty)$
	R: $[0, \infty)$	R: $[-3, \infty)$

You have seen that the inverses of functions are not necessarily functions. When both a relation and its inverse are functions, the relation is called a *one-to-one function*. In a <u>one-to-one function</u>, each y-value is paired with exactly one x-value.

Application

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The number of times that a cricket chirps per minute can be found by using the function $N(F) = 4F - 160$, where F is the temperature in degrees Fahrenheit.

(a) Find and interpret the inverse of $N(F)$.

(b) What is the temperature when the cricket is chirping 60 times a minute?

(c) How many times will the cricket chirp in 1 minute at a temperature of 80 ?