You can perform operations on functions in much the same way that you perform operations on numbers or expressions. You can add, subtract, multiply, or divide functions by operating on their rules.

Notation for Function Operations	
Operation	**Notation**
Addition	$(f+g)(x) = f(x) + g(x)$
Subtraction	$(f-g)(x) = f(x) - g(x)$
Multiplication	$(fg)(x) = f(x) \cdot g(x)$
Division	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, where $g(x) \neq 0$

Example 1A: Adding and Subtracting Functions

Given $f(x) = 4x^2 + 3x - 1$ and $g(x) = 6x + 2$, find each function.

$\boxed{(f + g)(x) = 4x^2 + 9x + 1}$

$= (4x^2 + 3x - 1) + (6x + 2)$

$\boxed{(f - g)(x) = 4x^2 - 3x - 3}$

$f(x) - g(x) = (4x^2 + 3x - 1) - (6x + 2)$

Check It Out! Example 1a

Given $f(x) = 5x - 6$ and $g(x) = x^2 - 5x + 6$, find each function.

$(f + g)(x)$ $= x^2$

$(f - g)(x)$ $= -x^2 + 10x - 12$

$5x - 6 - (x^2 - 5x + 6)$

When you divide functions, be sure to note any domain restrictions that may arise.

Example 2A: Multiplying and Dividing Functions

Given $f(x) = 6x^2 - x - 12$ and $g(x) = 2x - 3$, find each function.

$(fg)(x) = 12x^3 - 20x^2 - 21x + 36$

$(6x^2 - x - 12)(2x - 3)$

Example 2B: Multiplying and Dividing Functions

$$\left(\frac{f}{g}\right)(x) = \frac{6x^2 - x - 12}{2x - 3}$$

$$= \frac{(2x-3)(3x+4)}{(2x-3)}$$

$$\left(\frac{f}{g}\right)(x) = 3x + 4, \quad x \neq 3/2$$

Check It Out! Example 2a

Given $f(x) = x + 2$ and $g(x) = x^2 - 4$, find each function.

$(fg)(x)$ = $x^3 + 2x^2 - 4x - 8$

$\left(\frac{g}{f}\right)(x)$ = $x - 2, \; x \neq -2$

Another function operation uses the output from one function as the input for a second function. This operation is called the **composition of functions**.

Composition of Functions

The composition of functions *f* and *g* is notated

$$(f \circ g)(x) = f(g(x)).$$ "f of g of x"

The domain of $(f \circ g)(x)$ is all values of x in the domain of g such that $g(x)$ is in the domain of f.

Caution!

Be careful not to confuse the notation for multiplication of functions with composition

$fg(x) \neq f(g(x))$

$(f \cdot g)(x) \neq (f \circ g)(x)$

Example 3A: Evaluating Composite Functions

Given $f(x) = 2^x$ and $g(x) = 7 - x$, find each value.

$g(4) = 7 - 4 = 3$

$f(\boxed{g(4)}) = f(3) = 2^3 = 8$

$(f \circ g)(4)$

$g(\boxed{f(4)}) = g(16) = -9$

$f(4) = 2^4 = 16$

Check It Out! Example 3a

Given $f(x) = 2x - 3$ and $g(x) = x^2$, find each value.

$g(3) = 3^2 = 9$

$f(\boxed{g(3)}) = 15$

$f(9) =$

$g(f(3)) = 9$

Example 4A: Writing Composite Functions

Given $f(x) = x^2 - 1$ and $g(x) = \frac{x}{1-x}$, write each composite function. State the domain of each.

$\boxed{f(g(x)) = \frac{2x-1}{(1-x)^2}}$, $x \neq 1$

$$f\left(\frac{x}{1-x}\right) = \left(\frac{x}{1-x}\right)^2 - 1$$

$$= \frac{x^2}{(1-x)^2} - \frac{1(1-x)^2}{(1-x^2)} = \frac{x^2-(1-2x+x^2)}{(1-x)^2}$$

Example 4B: Writing Composite Functions

Given $f(x) = x^2 - 1$ and $g(x) = \frac{x}{1 - x}$, write each composite function. State the domain of each.

$g(\boxed{f(x)}) = \frac{x^2-1}{2-x^2}$, $x \neq \pm\sqrt{2}$

$g(x^2-1) = \frac{x^2-1}{1-(x^2-1)} = \frac{x^2-1}{2-x^2}$

Check It Out! Example 4a

Given $f(x) = 3x - 4$ and $g(x) = \sqrt{x} + 2$, write each composite. State the domain of each.

$f(g(x)) = 3\sqrt{x} + 2$, $x \geq 0$ OR $[0, \infty)$

$g(f(x)) = \sqrt{3x-4} + 2$, $[4/3, \infty)$

$3x - 4 \geq 0$