In previous lessons, you learned how to transform several types of functions. You can transform piecewise functions by applying transformations to each piece independently. Recall the rules for transforming functions given in the table.

Transformations of $f(x)$	
Horizontal Translation	**Vertical Translation**
$f(x) \rightarrow f(x - h)$ left for $h < 0$ right for $h > 0$	$f(x) \rightarrow f(x) + k$ down for $k < 0$ up for $k > 0$
Reflection Across *y*-axis	**Reflection Across *x*-axis**
$f(x) \rightarrow f(-x)$ The graph is reflected across the y-axis.	$f(x) \rightarrow -f(x)$ The graph is reflected across the x-axis.
Horizontal Stretch/Compression	**Vertical Stretch/Compression**
$f(x) \rightarrow f\left(\frac{1}{b}x\right)$ stretch for $b > 1$ compression for $0 < b < 1$	$f(x) \rightarrow af(x)$ stretch for $a > 1$ compression for $0 < a < 1$

Caution

Horizontal transformations change both the rules and the intervals of piecewise functions. Vertical transformations change only the rules.

Example 1: Transforming Piecewise Functions

Given $f(x) = \begin{cases} -\frac{1}{2}x & \text{if } x < 0 \\ \frac{1}{2}x^2 & \text{if } x \geq 0 \end{cases}$ write the rule $g(x)$, a vertical stretch by a factor of 3.

$g(x) = 3f(x)$

$g(x) = \begin{cases} -\frac{3}{2}x, & x < 0 \\ \frac{3}{2}x^2, & x \geq 0 \end{cases}$

Example 2: Transforming Piecewise Functions

$x \cdot 4$

Given $f(x) = \begin{cases} x + 3 & \text{if } x > 0 \\ 2x + 3 & \text{if } x \le 0 \end{cases}$ write the

$2(x-4)+3$

rule $g(x)$, a horizontal translation of $f(x)$ 4 units right.

$g(x) = f(x-4)$

$g(x) = \begin{cases} x - 1, & x > 4 \\ 2x - 5, & x \le 4 \end{cases}$

Check It Out! Example 1

$f(x)$ $g(x)$

$(1, 1) \longrightarrow (2, 1)$

Given $f(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ x - 3 & \text{if } x > 1 \end{cases}$ write the rule

for $g(x)$, a horizontal stretch of $f(x)$ by a factor of 2.

$g(x) = f\left(\frac{1}{2}x\right)$

$$g(x) = \begin{cases} \frac{1}{4}x^2, & x \leq 2 \\ \frac{1}{2}x - 3, & x > 2 \end{cases}$$

a) $g(x) = f(x+6)$

$$g(x) = \begin{cases} x+3, & x \le -6 \\ 4x+24, & x > -6 \end{cases}$$

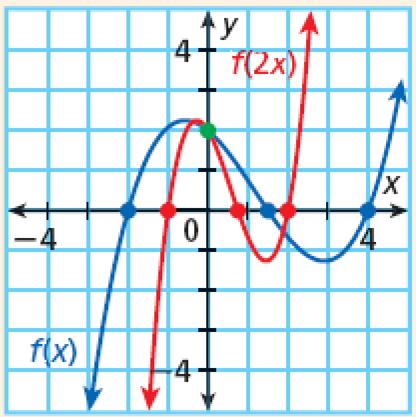
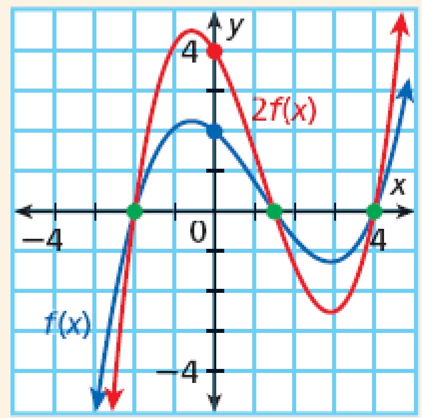
b) $h(x) = f(4x)$

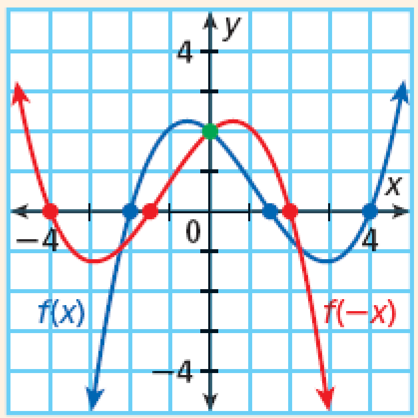
$$h(x) = \begin{cases} 4x-3, & x \le 0 \\ 16x, & x > 0 \end{cases}$$

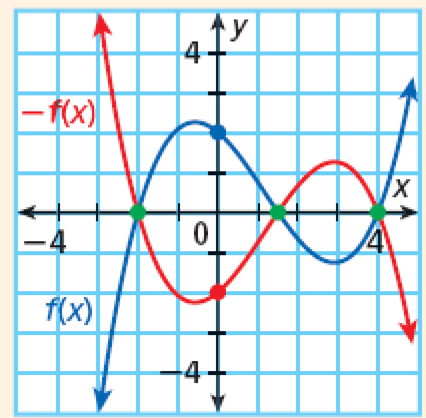
c) $p(x) = f(x) - 3$

$$p(x) = \begin{cases} x - 6, & x \le 0 \\ 4x - 3, & x > 0 \end{cases}$$

When functions are transformed, the intercepts may or may not change. By identifying the transformations, you can determine the intercepts, which can help you graph a transformed function.

Effects of Transformations on Intercepts of $f(x)$	
Horizontal Stretch or Compression by a Factor of *b*	**Vertical Stretch or Compression by a Factor of *a***
	
x-intercepts are multiplied by *b*. *y*-intercept stays the same.	*x*-intercepts stay the same. *y*-intercept is multiplied by *a*.

<table>
<tr><th>Reflection Across y-axis</th><th>Reflection Across x-axis</th></tr>
<tr><td>


</td><td>


</td></tr>
<tr><td>x-intercepts are negated.
y-intercept stays the same.</td><td>x-intercepts stay the same.
y-intercept is negated.</td></tr>
</table>

Example 3A: Identifying Intercepts

Identify the *x*- and *y*-intercepts of *f*(*x*). Without graphing *g*(*x*), identify its *x*- and *y*-intercepts.

$f(x) = -2x - 4$; $g(x) = f\left(\frac{1}{2}x\right)$

horizontal stretch by 2

x-int: (-2, 0)

y-int: (0, -4)

x-int: (-4, 0)

y-int: (0, -4)

Example 3B: Identify Intercepts

$f(x) = x^2 - 1$; $g(x) = f(-x)$ reflection in y-axis

x-int: (1,0)
(-1,0)

x-int: (-1,0)
(1,0)

y-int: (0,-1)

y-int: (0,-1)

Check It Out! Example 3A

Identify the *x*- and *y*-intercepts of *f*(*x*). Without graphing *g*(*x*), identify its *x*- and *y*-intercepts.

$f(x) = \frac{2}{3}x + 4$ and $g(x) = -f(x)$ reflection in x-axis

x-int: (-6, 0)

y-int: (0, 4)

x-int: (-6, 0)

y-int: (0, -4)

Check It Out! Example 2b

$f(x) = x^2 - 9$ and $g(x) = \frac{1}{3} f(x)$

vertical compression by 1/3

(3,0), (-3,0)

(0, -9)

(3,0), (-3,0)

(0, -3)

Example 4: Problem-Solving Application

Coco's Coffee charges different prices based on the number of pounds purchased. The pricing scale is modeled by the function below, where *w* is the weight in pounds purchased.

$$p(w) = \begin{cases} 9w & \text{if } 0 < w < 3 \\ 27 + 7.5(w-3) & \text{if } 3 \le w < 6 \\ 49.5 + 6(w-6) & \text{if } w \ge 6 \end{cases}$$

Example 4 Continued

Orders placed directly through the Web site are discounted by $\frac{1}{3}$, but a shipping fee of \$2.50 is added. Write a pricing function for orders placed through the Web site.

$$q(w) = \frac{2}{3}P(w) + 2.50$$

$$q(w) = \begin{cases} 6w + 2.5, & 0 < w < 3 \\ 20.5 + 5(w-3), & 3 \le w < 6 \\ 35.5 + 4(w-6), & w \ge 6 \end{cases}$$

$$\frac{2}{3}(9w) + 2.50$$

$$\frac{2}{3}\left(\underline{27} + \underline{7.5(w-3)}\right) + 2.50$$

$$18 + 5(w-3) + 2.50$$