6.3 Piecewise Functions

Piecewise Function: a function represented by a combination of one or more functions, each corresponding to a part of the domain

ex. $f(x)=\begin{cases}2x-1, x\le 1\\ 3x+1, x>1\end{cases}$

***Evaluating Piecewise Functions**

Ex. Evaluate the function at the indicated value.

1) $f(x)=\begin{cases}3x+2, x\le 3\\ x-1, x>3\end{cases}$

$f(0)=$ $3(0)+2=2$

$f(2)=$ $3(2)+2=8$

$f(20)=$ $20-1=19$

2) $f(x)=\begin{cases}x-7, x<1\\ 3x-5, x\ge 1\end{cases}$

$f(-1)=$ -8

$f(1)=$ -2

$f(0)=$ -7

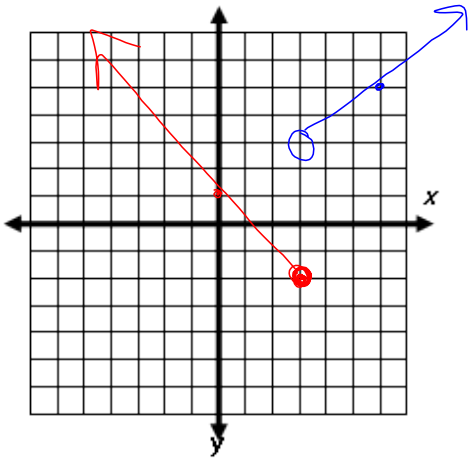
Graphing Piecewise Functions

Ex. Graph $f(x)=\begin{cases}-x+1, x\le 3\\ \frac{2}{3}x+1, x>3\end{cases}$

D: $(-\infty,\infty)$

R: $[-2,\infty)$

*evaluate each piece of the function for at least two values (include endpoints)



x	$-x+1$
3	-2
0	1

endpoint

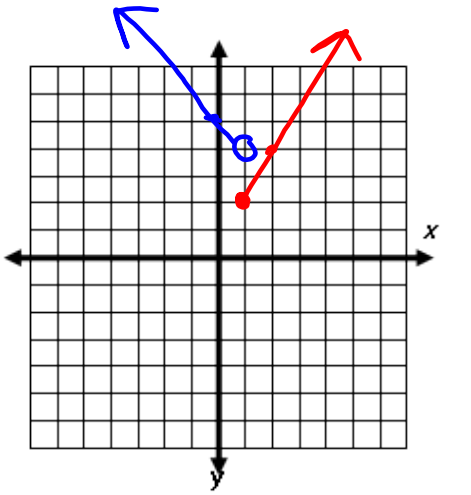
x	$\frac{2}{3}x+1$
3	3
6	5

Graphing Piecewise Functions

Ex. #2 Graph $f(x)=\begin{cases}-x+5, x<1\\ 2x, x\ge 1\end{cases}$

D: $(-\infty,\infty)$

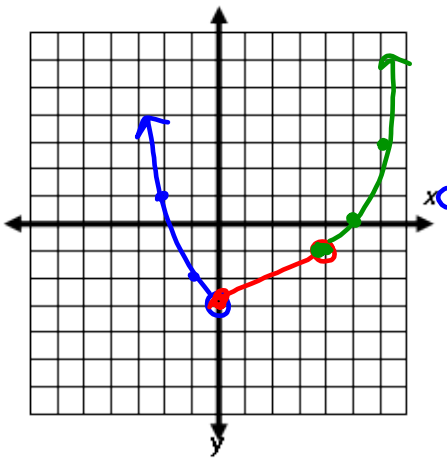
R: $[2,\infty)$



Graphing Piecewise Functions

Ex. #3 Graph $g(x) = \begin{cases} x^2 - 3 & \text{if } x < 0 \\ \frac{1}{2}x - 3 & \text{if } 0 \le x < 4 \\ (x-4)^2 - 1 & \text{if } x \ge 4 \end{cases}$

x	$(x-4)^2-1$
4	-1
5	0
6	3



x	x^2-3
0	-3
-1	-2
-2	1

D: $(-\infty, \infty)$

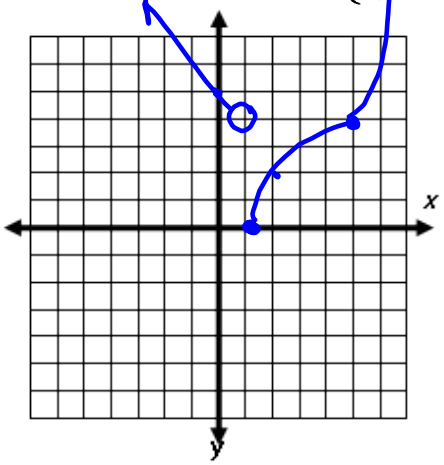
R: $[-3, \infty)$

x	$\frac{1}{2}x-3$
0	-3
4	-1

Graphing Piecewise Functions

Ex. #4 Graph $f(x) = \begin{cases} -x+5, x<1 \\ 2\sqrt{x-1}, 1 \le x < 5 \\ (x-3)^2, x \ge 5 \end{cases}$

1	0
2	2
5	4



D: $(-\infty, \infty)$

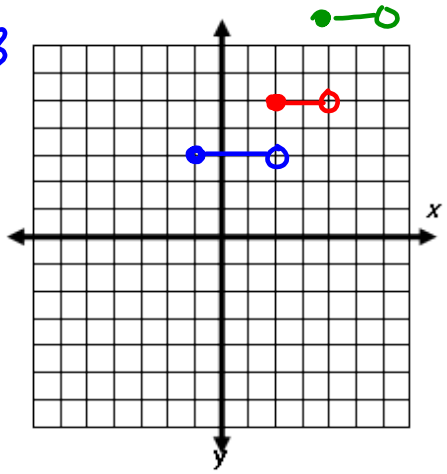
R: $[0, \infty)$

Graphing Piecewise Functions

Ex. #5 Graph $f(x) = \begin{cases} 3, -1 \le x < 2 \\ 5, 2 \le x < 4 \\ 8, 4 \le x < 6 \end{cases}$

**This is called a "step function" because it is constant for each interval of its domain.

x	3
-1	3
2	3



D: [-1, 6)

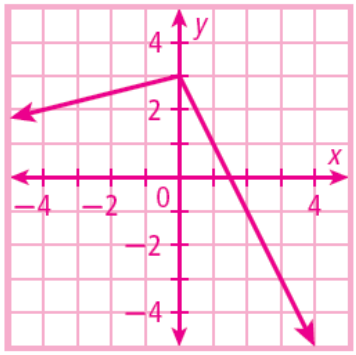
R: {3, 5, 8}

Notice that piecewise functions are not necessarily *continuous*, meaning that the graph of the function may have breaks or gaps.

To write the rule for a piecewise function, determine where the domain is divided and write a separate rule for each piece. Combine the pieces by using the correct notation.

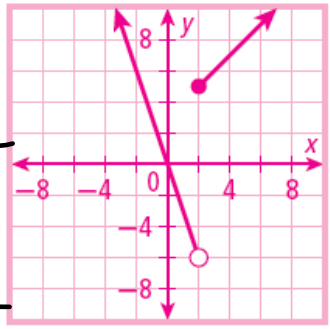
Ex #6 Write a piecewise function for the graph.

$$f(x) = \begin{cases} \frac{1}{4}x + 3, & x \le 0 \\ -2x + 3, & x > 0 \end{cases}$$



Ex #7 Write a piecewise function for the graph.

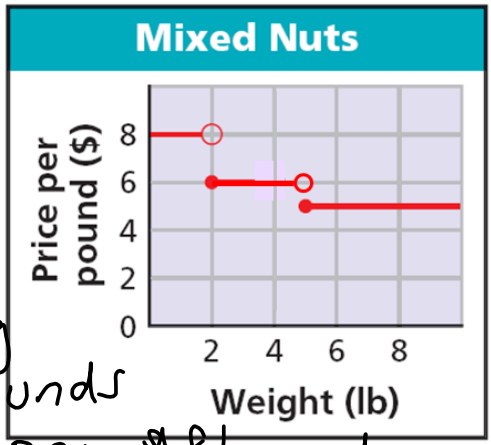
$$f(x) = \begin{cases} -3x, x < 2 \\ x + 3, x \geq 2 \end{cases}$$



Ex #8 Application

Create a table and a verbal description to represent the graph.

Weight	price/pound
$0 < w < 2$	$8
$2 \leq w < 5$	$6
$w \geq 5$	$5



If you're buying less than 2 pounds of nuts you'll pay $8/pound.

If you buy 2 or more pounds but less than 5, you'll pay $6/pound.

Ex #9 Application

You have a summer job that pays time and a half for overtime (working more than 40 hours). After that, you earn 1.5 times your hourly rate of $7.00/hr.

Write and graph a piecewise function that gives your weekly pay, *P,* in terms of the number hours you work *h.*

How much will you make if you work 45 hours?

$$P(h)=\begin{cases}7h, & 0\le h\le 40\\ 10.5h-140, & h>40\end{cases}$$

$7(40)+10.50(h-40)$

$280+10.5h-420$

$$P(h)=\begin{cases}7h, \text{ if } 0\le h\le 40\\ 10.5h-140, \text{ if } h>40\end{cases}$$

Ex #10 Application

You are employed by a company in which commission rates are based on how much you sell. If you sell up to $100,000 of merchandise in a month, you earn 5% of sales as a commission. If you sell over $100,000, you earn 8% commission on your sales.

A) Write a piecewise function that gives the amount you earn, *C*, in commission in a given month for *x* dollars in sales.

B) How much will you earn if you sell $165,000 of merchandise?

$$C(x) = \begin{cases} 0.05x, & \text{if } 0 < x \leq 100{,}000 \\ 0.08x, & \text{if } x > 100{,}000 \end{cases}$$