

5-6 Radical Expressions and Rational Exponents

A **rational exponent** is an exponent that can be expressed as $\frac{m}{n}$, where m and n are integers and $n \neq 0$. Radical expressions can be written by using rational exponents.

Rational Exponents

For any natural number n and integer m ,

WORDS	NUMBERS	ALGEBRA
The exponent $\frac{1}{n}$ indicates the n th root.	$16^{\frac{1}{4}} = \sqrt[4]{16} = 2$	$a^{\frac{1}{n}} = \sqrt[n]{a}$
The exponent $\frac{m}{n}$ indicates the n th root raised to the m th power.	$8^{\frac{2}{3}} = (\sqrt[3]{8})^2 = 2^2 = 4$	$a^{\frac{m}{n}} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$

5-6 Radical Expressions and Rational Exponents

Example 1: Writing Expressions in Radical Form

Write the expression $(-32)^{\frac{3}{5}}$ in radical form and simplify.

$$\begin{aligned} & (\sqrt[5]{-32})^3 \quad \text{OR} \quad \sqrt[5]{(-32)^3} \\ & (-2)^3 \\ & (-8) \end{aligned}$$

5-6 Radical Expressions and Rational Exponents

Check It Out! Example 1a

Write the expression $4^{\frac{5}{2}}$ in radical form, and simplify.

$$(\sqrt{4})^5 = 2^5 = 32$$

5-6 Radical Expressions and Rational Exponents

Check It Out! Example 1b

Write the expression $625^{\frac{3}{4}}$ in radical form, and simplify.

$$(\sqrt[4]{625})^3 = 5^3 = 125$$

5-6 Radical Expressions and Rational Exponents

Example 2: Writing Expressions by Using Rational Exponents

Write each expression by using rational exponents.

A. $\sqrt[8]{13^4} = 13^{\frac{4}{8}} = 13^{\frac{1}{2}}$

B. $\sqrt[5]{13^{15}} = 13^3 = 2197$

5-6 Radical Expressions and Rational Exponents

Check It Out! Example 3

Write each expression by using rational exponents.

a. $(\sqrt[4]{81})^3$

b. $\sqrt[3]{10^9}$

c. $\sqrt[4]{5^2}$

5-6 Radical Expressions and Rational Exponents

Example 4

Simplify each expression.

a. $7^{\frac{7}{9}} \cdot 7^{\frac{11}{9}}$

$$7^{\frac{18}{9}} = 7^2$$

$$\text{(49)}$$

b. $\frac{16^{\frac{3}{4}}}{16^{\frac{5}{4}}}$

$$= 16^{-\frac{2}{4}} = 16^{-\frac{1}{2}}$$

$$= \frac{1}{\sqrt{16}} = \frac{1}{4}$$

c. $(-8)^{-\frac{1}{3}}$

$$= \frac{1}{(-8)^{\frac{1}{3}}}$$

$$= \frac{1}{\sqrt[3]{-8}} = \frac{1}{-2}$$

Holt McDougal Algebra 2

Copyright © by Holt Mc Dougal. All Rights Reserved.

5-6 Radical Expressions and Rational Exponents

Example 4

Simplify each expression. Your answer should contain only positive exponents with no fractional exponents in the denominator.

d. $x^{\frac{1}{4}} \cdot x^{\frac{7}{3}}$

$$x^{\frac{3}{12}} \cdot x^{\frac{28}{12}}$$

$$x^{\frac{31}{12}}$$

e. $\left(\frac{x^{3/5}}{81}\right)^{1/4}$

$$= \frac{(x^{3/5})^{1/4}}{81^{1/4}}$$

$$= \frac{x^{3/20}}{3}$$

f. $\frac{3y^{2/5}}{9y^{1/2}}$

$$= \frac{1y^{4/10}}{3y^{5/10}}$$

$$= \frac{1}{3} y^{-1/10} = \frac{1y^{-1/10}}{3}$$

$$= \frac{1}{3y^{1/10}} \cdot \frac{y^{9/10}}{y^{9/10}} = \frac{y^{8/10}}{3y}$$

Holt McDougal Algebra 2

Copyright © by Holt Mc Dougal. All Rights Reserved.

5-6 Radical Expressions and Rational Exponents

Example 4

Simplify each expression. Your answer should contain only positive exponents with no fractional exponents in the denominator.

$$\frac{x^5}{x^2} = x^3$$

$$\frac{x^2}{x^5} = \frac{1}{x^3}$$

g. $\frac{6x^{3/4}}{12x^{-1/2}}$

$$\frac{x^{5/4}}{2}$$

h. $\frac{x^{3/5}}{x^{2/9}}$

$$x^{17/45}$$

$$\frac{(343xy^2)^{1/3}(49x^{3/5}y)^{3/2}}{14x^{4/7}y^{8/3}}$$