5-6 Radical Expressions and Rational Exponents

We are already familiar with finding the square root of a number. Squaring and square rooting are inverses of each other. There are also roots that correspond to larger powers.

5 and –5 are square roots of 25 because $5^2 = 25$ and $(–5)^2 = 25$

2 is the cube root of 8 because $2^3 = 8$.

2 and –2 are fourth roots of 16 because $2^4 = 16$ and $(–2)^4 = 16$.

* a is the *n*th root of *b* if $a^n = b$.

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The *n*th root of a real number *a* can be written as the radical expression $\sqrt[n]{a}$, where *n* is the <u>index</u> (plural: *indices*) of the radical and *a* is the radicand.

When a number has more than one root, the radical sign indicates only the principal, or positive, root.

When a radical sign shows no index, it represents a square root.

Numbers and Types of Real Roots

Case	Roots	Example
Odd index	1 real root	The real 3rd root of 8 is 2.
Even index; positive radicand	2 real roots	The real 4th roots of 16 are ±2.
Even index; negative radicand	0 real roots	–16 has no real 4th roots.
Radicand of 0	1 root of 0	The 3rd root of 0 is 0.

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Example 1: Finding Real Roots

Find all real roots.

A. sixth roots of 64 2, -2

$(2)^6 = 64$

$(-2)^6 = 64$

A positive number has two real sixth roots. Because $2^6 = 64$ and $(-2)^6 = 64$, the roots are 2 and –2.

B. cube roots of –216 $= -6$

$(-6)^3 = -216$

A negative number has one real cube root. Because $(-6)^3 = -216$, the root is –6.

C. fourth roots of –1024 not a thing

no real roots

A negative number has no real fourth roots.

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The properties of square roots you already know also apply to *n*th roots.

Keep in mind that in order **for a radical expression to be *completely* simplified, the following must be true:**

- **For an *n*th root, the radicand has no factors that are perfect *n*ths.**
- **There are no fractions in the radicand.**
- **There are no radicals in the denominator.**

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Simplifying Radical

(1) $\sqrt{24} = \sqrt{4} \cdot \sqrt{6} = 2\sqrt{6}$

(2) $\sqrt[3]{-162} = \sqrt[3]{-27} \cdot \sqrt[3]{6} = \boxed{-3\sqrt[3]{6}}$

(5) $\sqrt[4]{128n^8} = \sqrt[4]{16} \cdot \sqrt[4]{8} \cdot \sqrt[4]{n^8}$

$= \boxed{2n^2 \sqrt[4]{8}}$

$(?)^4 = n^8$

$(n^2)^4 = n^8$

5-6 Radical Expressions and Rational Exponents

$\sqrt[5]{r^5 r^2}$

(7) $\sqrt[5]{224r^7} = \sqrt[5]{32} \cdot \sqrt[5]{7} \cdot \sqrt[5]{r^5} \cdot \sqrt[5]{r^2}$

$2r\sqrt[5]{7r^2}$

(13) $\sqrt[4]{128x^7y^9} = \sqrt[4]{16} \cdot \sqrt[4]{8} \cdot \sqrt[4]{x^4} \cdot \sqrt[4]{x^3} \cdot \sqrt[4]{y^8} \cdot \sqrt[4]{y}$

$2xy^2\sqrt[4]{8x^3y}$

5-6 Radical Expressions and Rational Exponents

(5) $\sqrt[6]{448x^7y^{17}} = 2xy^2\sqrt[6]{7xy^5}$

Adding, Subtr, Mult.

9) $-3\sqrt[3]{-3} + 2\sqrt[3]{162} + 3\sqrt[3]{81}$

$-3\sqrt[3]{-1}\cdot\sqrt[3]{3} \quad 2\cdot\sqrt[3]{27}\cdot\sqrt[3]{6} + 3\sqrt[3]{27}\cdot\sqrt[3]{3}$

$3\sqrt[3]{3} + 6\sqrt[3]{6} + 9\sqrt[3]{3}$

$12\sqrt[3]{3} + 6\sqrt[3]{6}$

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5-6 Radical Expressions and Rational Exponents

Dividing

(21) $\dfrac{\sqrt[4]{5}}{4\sqrt[4]{27}} \cdot \dfrac{\sqrt[4]{3}}{\sqrt[4]{3}} = \dfrac{\longrightarrow}{4\sqrt[4]{81}} = \dfrac{\sqrt[4]{15}}{12}$

perfect fourth

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