5-4 Rational Functions

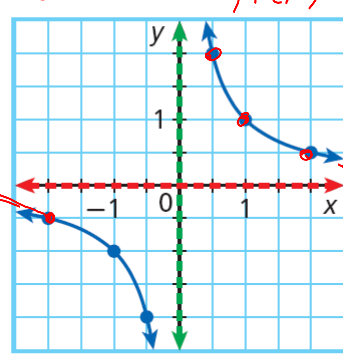
A **rational function** is a function whose rule can be written as a ratio of two polynomials.

The parent rational function is $f(x) = \frac{1}{x}$. Its graph is a *hyperbola*, which has two separate branches. You will learn more about hyperbolas later in this course.

5-4 Rational Functions

as $x \to \infty, f(x) \to 0$
as $x \to -\infty, f(x) \to 0$

The function $f(x) = \frac{1}{x}$ has a vertical asymptote at $x = 0$ and a horizontal asymptote at $y = 0$.



x	$\frac{1}{x}$
-2	$-\frac{1}{2}$
0	undefined
1	1
2	$\frac{1}{2}$
$\frac{1}{2}$	2

5-4 Rational Functions

The rational function $f(x) = \frac{1}{x}$ can be transformed by using methods similar to those used to transform other types of functions.

$|a|$ ⟶ vertical stretch or compression factor
$a < 0$ ⟶ reflection across the x-axis

$$f(x) = \frac{a}{x - h} + k$$

k ⟶ vertical translation
down for $k < 0$; up for $k > 0$

h ⟶ horizontal translation
left for $h < 0$; right for $h > 0$

5-4 Rational Functions

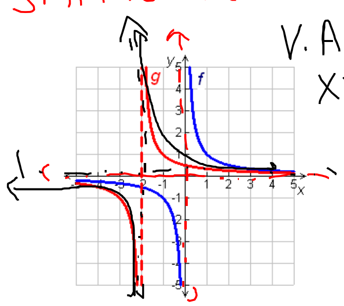
Example 1: Transforming Rational Functions

Using the graph of $f(x) = \frac{1}{x}$ as a guide, describe the transformation and graph each function.

A. $g(x) = \frac{1}{x + 2}$

Shifts left 2

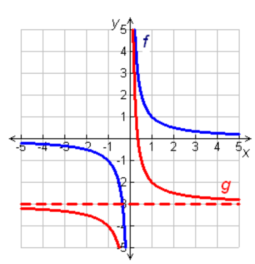
V.A.
x = -2



B. $g(x) = \frac{1}{x} - 3$

down 3

HA:
y = -3



5-4 Rational Functions

Check It Out! Example 1

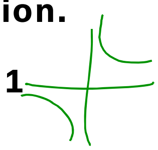
Using the graph of $f(x) = \frac{1}{x}$ as a guide, describe the transformation and graph each function.

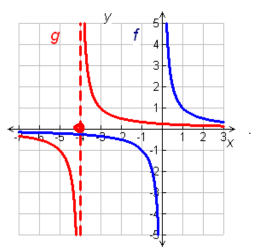
a. $g(x) = \frac{1}{x+4}$

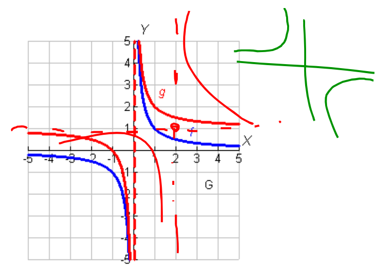
$(h = -4$

$k = 0$

b. $g(x) = \frac{1}{x-2} + 1$







5-4 Rational Functions

The values of *h* and *k* affect the locations of the asymptotes, the domain, and the range of rational functions whose graphs are hyperbolas.

Rational Functions

For a rational function of the form $f(x) = \frac{a}{x-h} + k$,

- the graph is a hyperbola.
- there is a vertical asymptote at the line $x = h$, and the domain is $\{x \mid x \neq h\}$. $(-\infty, h) \cup (h, \infty)$
- there is a horizontal asymptote at the line $y = k$, and the range is $\{y \mid y \neq k\}$. $(-\infty, k) \cup (k, \infty)$

5-4 Rational Functions

Example 2: Determining Properties of Hyperbolas

Identify the asymptotes, domain, and range of the function $g(x) = \frac{1}{x+3} - 2$.

Vertical asymptote: $x = -3$

Domain: $x \neq -3$

Horizontal asymptote: $y = -2$

Range: $y \neq -2$

Check Graph the function on a graphing calculator.

5-4 Rational Functions

Check It Out! Example 2

Identify the asymptotes, domain, and range of the function $g(x) = \frac{1}{x-3} - 5$.

Vertical asymptote:

Domain:

Horizontal asymptote:

Range:

Check Graph the function on a graphing calculator.

5-4 Rational Functions

A **discontinuous function** is a function whose graph has one or more gaps or breaks. The hyperbola graphed in Example 2 and many other rational functions are discontinuous functions.

A **continuous function** is a function whose graph has no gaps or breaks. The functions you have studied before this, including linear, quadratic, and polynomial are continuous functions.

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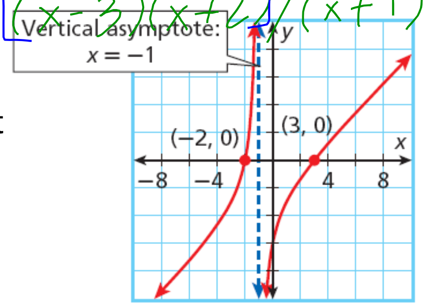
The graphs of some rational functions are not hyperbolas. Consider the rational function $f(x) = \frac{(x-3)(x+2)}{x+1}$ and its graph.

$y = [(x-3)(x+2)]/(x+1)$

The numerator of this function is 0 when $x = 3$ or $x = -2$. Therefore, the function has x-intercepts at -2 and 3. The denominator of this function is 0 when $x = -1$. As a result, the graph of the function has a vertical asymptote at the line $x = -1$.



5-4 Rational Functions

Zeros and Vertical Asymptotes | **Rational Functions**

If $f(x) = \frac{p(x)}{q(x)}$, where p and q are polynomial functions in standard form with no common factors other than 1, then the function f has

- zeros at each real value of x for which $p(x) = 0$.
- a vertical asymptote at each real value of x for which $q(x) = 0$.

5-4 Rational Functions

Example 3: Graphing Rational Functions with Vertical Asymptotes

Identify the zeros and vertical asymptotes of $f(x) = \frac{(x^2 + 3x - 4)}{x + 3}$.

Step 1 Find the zeros and vertical asymptotes.

$f(x) = \frac{(x + 4)(x - 1)}{x + 3}$ — *Factor the numerator.*

Zeros: –4 and 1 — *The numerator is 0 when $x = -4$ or $x = 1$.*

Vertical asymptote: $x = -3$ — *The denominator is 0 when $x = -3$.*

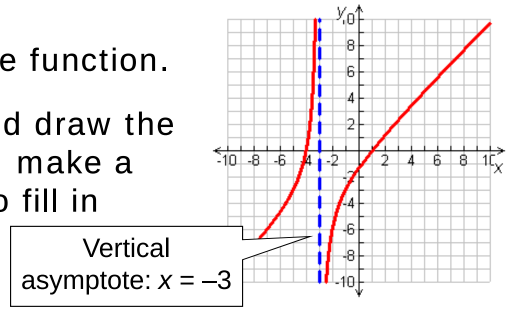
5-4 Rational Functions

Example 3 Continued

Identify the zeros and vertical asymptotes of $f(x) = \dfrac{(x^2 + 3x - 4)}{x + 3}$.

Step 2 Graph the function.

Plot the zeros and draw the asymptote. Then make a table of values to fill in missing points.



x							
y							

5-4 Rational Functions

Check It Out! Example 3

Identify the zeros and vertical asymptotes of $f(x) = \dfrac{(x^2 + 7x + 6)}{x + 3}$.

Step 1 Find the zeros and vertical asymptotes.

$f(x) = \dfrac{(x + 6)(x + 1)}{x + 3}$ *Factor the numerator.*

Zeros: –6 and –1 *The numerator is 0 when $x = -6$ or $x = -1$.*

Vertical asymptote: $x = -3$ *The denominator is 0 when $x = -3$.*

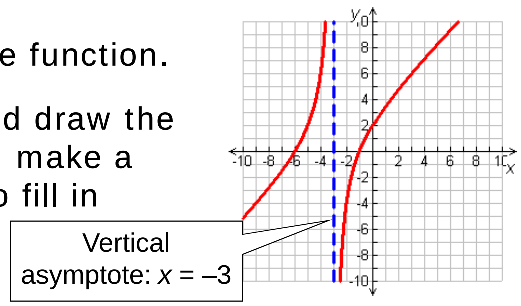
5-4 Rational Functions

Check It Out! Example 3 Continued

Identify the zeros and vertical asymptotes of $f(x) = \frac{(x^2 + 7x + 6)}{x + 3}$.

Step 2 Graph the function.

Plot the zeros and draw the asymptote. Then make a table of values to fill in missing points.



x							
y							