

5.3 Adding/Subtracting Rational Expressions

- To add/subtract rational expressions, the expressions must have a common denominator (preferably the least common denominator). The LCD must account for every factor of every denominator.
- Add/subtract numerators only - do NOT change the denominator.
- Don't forget to SIMPLIFY your final answer.

$$\textcircled{1} \quad \frac{3}{x^2 - 4} + \frac{1}{x + 2}$$

$$\text{LCD: } (x+2)(x-2)$$

$$\frac{3}{(x+2)(x-2)} + \frac{1}{(x+2)} \frac{(x-2)}{(x-2)} = \frac{3}{\text{LCD}} + \frac{x-2}{\text{LCD}} = \frac{x+1}{(x+2)(x-2)}$$

$\textcircled{2}$

$$\frac{1}{x^2 - 3x + 2} - \frac{1}{x^2 - 4}$$

$$\text{LCD: } (x-2)(x-1)(x+2)$$

$$\frac{1}{(x-2)(x-1)} \frac{(x+2)}{(x+2)} - \frac{1}{(x+2)(x-2)} \frac{(x-1)}{(x-1)} = \frac{x+2}{\text{LCD}} - \frac{(x-1)}{\text{LCD}} = \frac{3}{(x-2)(x-1)(x+2)}$$

Try these on your own.

$$\textcircled{3} \frac{5}{4x+8} + \frac{3x}{x^2+4x+4}$$

LCD: $4(x+2)(x+2)$

$$\frac{5(x+2)}{4(x+2)(x+2)} + \frac{(3x) \cdot 1}{(x+2)(x+2)} = \frac{5x+10}{LCD} + \frac{12x}{LCD} = \frac{17x+10}{4(x+2)(x+2)}$$

$$\textcircled{4} \frac{1}{x} + \frac{3}{x+2} - \frac{7}{x^2+5x+6} = \frac{4x^2+7x+6}{x(x+2)(x+3)}$$

$$\textcircled{5} \frac{3}{4xy^2} - \frac{7}{8x^3y} = \frac{6x^2-7y}{8x^3y^2}$$

Some rational expressions are *complex fractions*.

A **complex fraction** contains one or more fractions in its numerator, its denominator, or both. Examples of complex fractions are shown below.

$$\frac{x+2}{\frac{3}{x}}$$

$$\frac{1 + \frac{1}{x}}{4x+5}$$

$$\frac{\frac{x+3}{x}}{\frac{x+4}{7x}}$$

Recall that the bar in a fraction represents division. Therefore, you can rewrite a complex fraction as a division problem and then simplify. You can also simplify complex fractions by using the LCD of the fractions in the numerator and denominator.

Examples

Simplify. Assume that all expressions are defined.

$$(5) \frac{\frac{20}{x-1}}{\frac{6}{3x-3}} = \frac{20}{x-1} \div \frac{6}{3x-3} = \frac{20}{x-1} \cdot \frac{3(x-1)}{6 \cancel{3}} = 10$$

$$(6) \frac{\frac{3}{x} + \frac{x}{2}}{\frac{x-1}{x}} = \left(\frac{3}{2x} + \frac{x^2}{2x} \right) \cdot \frac{x}{x-1} = \frac{x^2+6}{2x} \cdot \frac{x}{x-1} = \frac{x^2+6}{2(x-1)}$$

$$(7) \frac{\frac{x+1}{x^2-1}}{\frac{x}{x-1}} = \frac{1}{x}$$

$$(8) \frac{\frac{1}{x} + \frac{1}{2x}}{\frac{x+4}{x-2}}$$