Variation Functions 5-1

You've studied many types of linear functions. One special type of linear function is called *direct* variation.

A direct variation is a relationship between two variables x and y that can be written in the form y = kx, where $k \neq 0$.

In this relationship, *k* is the **constant of <u>variation</u>**. For the equation y = kx, y varies directly as x.

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5-1 **Variation Functions**

A direct variation equation is a linear equation in the form y = mx + b, where b = 0 and the constant of variation k is the slope. Because b = 0, the graph of a direct variation always passes through the origin.

Variation Functions 5-1)

Example 1: Writing and Graphing Direct Variation

Given: y varies directly as x, and y = 27 when x = 6. Write the direct variation function.

y = kx 27 = $k(6)$	y varies directly as x. Substitute 27 for y and 6 for x.
k = 4.5 $y = 4.5x$	Solve for the constant of variation k. Write the variation function by using the value of k.
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5-1 **Variation Functions**

Check It Out! Example 1

Given: y varies directly as x_1 and y = 6.5 when x = 13. Write the direct variation function.

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When you want to find specific values in a direct variation problem, you can solve for k and then use substitution or you can use the proportion derived below.

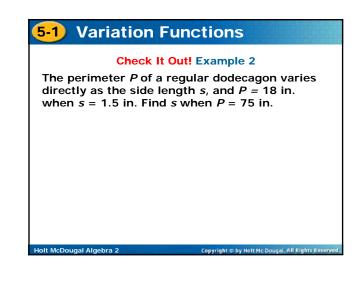
$$y_1 = kx_1 \rightarrow \frac{y_1}{x_1} = k$$
 and $y_2 = kx_2 \rightarrow \frac{y_2}{x_2} = k$ so, $\frac{y_1}{x_1} = \frac{y_2}{x_2}$.
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5-1 Variation Functions

Example 2: Solving Direct Variation Problems

The cost of an item in euros e varies directly as the cost of the item in dollars d, and e = 3.85 euros when d = \$5.00. Find dwhen e = 10.00 euros. Method 1 Find k. e = kd3.85 = k(5.00)Substitute. 0.77 = kSolve for k. Write the variation function. e = 0.77dUse 0.77 for k. 10.00 = 0.77dSubstitute 10.00 for e. 12.99 $\approx d$ Solve for d. Holt McDr al All R

5-1 Variation Functions		
Example 2 Continued		
Method 2 Use a proportion.		
$\frac{e_1}{d_1} = \frac{e_2}{d_2}$		
$\frac{3.85}{5.00} = \frac{10.00}{d}$	Substitute.	
3.85d = 50.00	Find the cross products.	
[∲] 12.99 ≈ d	Solve for d.	
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A <u>joint variation</u> is a relationship among three variables that can be written in the form y = kxz, where k is the constant of variation. For the equation y = kxz, y varies jointly as x and z.

5-1) Variation Functions

Example 3: Solving Joint Variation Problems

The volume V of a cone varies jointly as the area of the base B and the height h, and $V = 12\pi$ ft³ when $B = 9\pi$ ft³ and h = 4 ft. Find B when $V = 24\pi$ ft³ and h = 9 ft. Step 1 Find k. Step 2 Use the variation V = kBh function.

 $12\pi = k(9\pi)(4)$ Substitute. $V = \frac{1}{3}Bh$ Use $\frac{1}{3}$ for k $\frac{1}{3} = k$ Solve for k. $24\pi = \frac{1}{3}B(9)$ Substitute. $8\pi = B$ Solve for B. The base is 8π ft².

5-1 Variation Functions

Check It Out! Example 3

The lateral surface area *L* of a cone varies jointly as the area of the base radius *r* and the slant height *I*, and $L = 63\pi \text{ m}^2$ when r = 3.5 mand I = 18 m. Find *r* to the nearest tenth when $L = 8\pi \text{ m}^2$ and I = 5 m.

5-1) Variation Functions

A third type of variation describes a situation in which one quantity increases and the other decreases. For example, the table shows that the time needed to drive 600 miles decreases as speed increases. This type of variation is an inverse

on in v increases	Speed (mi/h)	Time (h)	Distance (mi)
eases. For	30	20	600
shows	40	15	600
ed to	50	12	600
creases			

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This type of variation is an inverse variation. An **inverse variation** is a relationship between two variables *x* and *y* that can be written in the form $y = \frac{k}{x}$, where $k \neq 0$. For the equation $y = \frac{k}{x}$, *y* varies inversely as *x*.

5-1 Variati	on Functions
Example 4: Writ	ing and Graphing Inverse Variation $\gamma = \frac{1}{2}$
	inversely as x_i and $y = 4$ ite the inverse variation
$y = \frac{k}{x}$	y varies inversely as x.
$4 = \frac{k}{5}$	<i>Substitute 4 for y and 5 for x.</i>
$k = 20$ $y = \frac{20}{x}$	Solve for k. Write the variation
Holt McDougal Algebra 2	formula.
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Check It Out! Example 4

Given: *y* varies inversely as *x*, and y = 4 when x = 10. Write the inverse variation function.

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5-1 Variation Functions When you want to find specific values in an inverse variation problem, you can solve for k and then use substitution or you can use the equation derived below. $y_1 = \frac{k}{x_1} \rightarrow y_1 x_1 = k$ and $y_2 = \frac{k}{x_2} \rightarrow y_2 x_2 = k$ so, $y_1 x_1 = y_2 x_2$.

5-1 Variation Functions			
Example 5: Sports Application $t = \frac{k}{5}$			
The time <i>t</i> needed to complete a certain race varies inversely as the runner's average speed <i>s</i>			
If a runner with an average speed of 8.82 mi/h completes the race in 2.97 h, what is the average speed of a runner who completes the race in 3.5 h?			
Method 1 Find k.	$t = \frac{k}{s}$ $2.97 = \frac{s}{8.82}$	Substitute.	
	k = 26.1954	Solve for k.	
	$t = \frac{26.1954}{s}$	Use 26.1954 for k.	
	$3.5 = \frac{26.1954}{1000}$	Substitute 3.5 for t.	
	$s \approx 7.48$	Solve for s.	
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Example 5 Continued

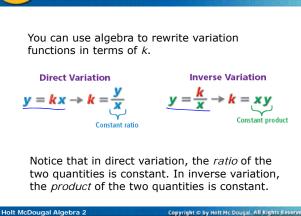
Method Use $t_1s_1 = t_2s_2$.	
$t_1 s_1 = t_2 s_2$	
(2.97)(8.82) = 3.5s	Substitute.
26.1954 = 3.5 <i>s</i>	Simplify.
7.48 ≈ <i>s</i>	Solve for s.
So the average speed of a rur	ner who compl

So the average speed of a runner who completes the race in 3.5 h is approximately 7.48 mi/h.

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t it takes f		
use varies v. If 20 vol ig hours, h	inversely unteers ca ow many	working hours
1	<i>v</i> . If 20 vol ng hours, h	v. If 20 volunteers can be

5-1 Variation Functions



Example 6: Identifying Direct and Inverse Variation Determine whether each data set represents a direct variation, an inverse variation, or neither. In each case xy = 52. Α. x 6.5 13 104 The product is constant, 0.5 8 4 so this represents an V inverse variation. В. In each case $\underline{y} = 6$. The 5 12 X 8 ratio is constant, so this 30 48 72 V represents a direct variation.

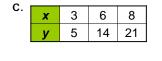
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Variation Functions

5-1

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Example 6: Identifying Direct and Inverse Variation Determine whether each data set represents a direct variation, an inverse variation, or neither.

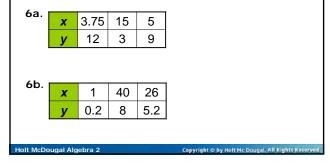


Since xy and $-\frac{y}{2}$ are not constant, this is neither a direct variation nor an inverse variation.



Check It Out! Example 6

Determine whether each data set represents a direct variation, an inverse variation, or neither.



5-1 Variation Functions

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A <u>combined variation</u> is a relationship that contains both direct and inverse variation. Quantities that vary directly appear in the numerator, and quantities that vary inversely appear in the denominator.

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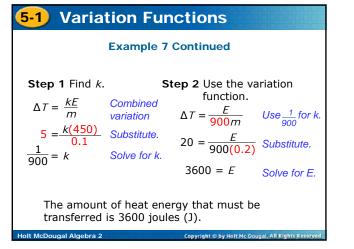
5-1) Variation Functions

Example 7: Chemistry Application

The change in temperature of an aluminum wire varies inversely as its mass m and directly as the amount of heat energy E transferred. The temperature of an aluminum wire with a mass of 0.1 kg rises 5°C when 450 joules (J) of heat energy are transferred to it. How much heat energy must be transferred to an aluminum wire with a mass of 0.2 kg raise its temperature 20°C?

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5-1 Variation Function	S	
direct : $y = k \times \frac{k}{x}$ inversely: $y = \frac{k}{x}$ Check It Out! Examp	le 7 $V = \frac{kT}{P}$	
The volume V of a gas varies inversely as the pressure P and directly as the temperature T.		
A certain gas has a volume of 10 liters (L), a		
temperature of 300 kelvins (K), and a pressure of 1.5 atmospheres (atm). If the gas		
is heated to 400K, and has a pre atm, what is its volume?		
V = 10 - 10 = k(300)	V=0.05(400)	
T= 300K 1.5		
p=1.3etm 15=K(300)	(V=20 L)	
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