5-1 Variation Functions

You've studied many types of linear functions. One special type of linear function is called *direct variation*.

A **direct variation** is a relationship between two variables x and y that can be written in the form $y = kx$, where $k \neq 0$.

In this relationship, k is the **constant of variation**. For the equation $y = kx$, y varies directly as x.

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A direct variation equation is a linear equation in the form $y = mx + b$, where $b = 0$ and the constant of variation k is the slope. Because $b = 0$, the graph of a direct variation always passes through the origin.

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Example 1: Writing and ~~Graphing~~ Direct Variation

Given: y varies directly as x, and $y = 27$ when $x = 6$. Write the direct variation function.

$y = kx$	*y varies directly as x.*
$27 = k(6)$	*Substitute 27 for y and 6 for x.*
$k = 4.5$	*Solve for the constant of variation k.*
$y = 4.5x$	*Write the variation function by using the value of k.*

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Check It Out! Example 1

Given: y varies directly as x, and $y = 6.5$ when $x = 13$. Write the direct variation function.

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When you want to find specific values in a direct variation problem, you can solve for k and then use substitution or you can use the proportion derived below.

$$y_1 = kx_1 \rightarrow \frac{y_1}{x_1} = k \quad \text{and} \quad y_2 = kx_2 \rightarrow \frac{y_2}{x_2} = k \quad \text{so,} \quad \frac{y_1}{x_1} = \frac{y_2}{x_2}.$$

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Example 2: Solving Direct Variation Problems

The cost of an item in euros *e* varies directly as the cost of the item in dollars *d*, and *e* = 3.85 euros when *d* = $5.00. Find *d* when *e* = 10.00 euros.

Method 1 Find k.

$e = kd$	
$3.85 = k(5.00)$	*Substitute.*
$0.77 = k$	*Solve for k.*

Write the variation function.

$e = 0.77d$	*Use 0.77 for k.*
$10.00 = 0.77d$	*Substitute 10.00 for e.*
$12.99 \approx d$	*Solve for d.*

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Example 2 Continued

Method 2 Use a proportion.

$\frac{e_1}{d_1} = \frac{e_2}{d_2}$	
$\frac{3.85}{5.00} = \frac{10.00}{d}$	*Substitute.*
$3.85d = 50.00$	*Find the cross products.*
\$ $12.99 \approx d$	*Solve for d.*

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Check It Out! Example 2

The perimeter *P* of a regular dodecagon varies directly as the side length *s*, and *P* = 18 in. when *s* = 1.5 in. Find *s* when *P* = 75 in.

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A **joint variation** is a relationship among three variables that can be written in the form $y = kxz$, where k is the constant of variation. For the equation $y = kxz$, y varies jointly as x and z.

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Example 3: Solving Joint Variation Problems

The volume V of a cone varies jointly as the area of the base B and the height h, and $V = 12\pi$ ft³ when $B = 9\pi$ ft² and $h = 4$ ft. Find B when $V = 24\pi$ ft³ and $h = 9$ ft.

Step 1 Find k.

$V = kBh$

$12\pi = k(9\pi)(4)$ *Substitute.*

$\frac{1}{3} = k$ *Solve for k.*

Step 2 Use the variation function.

$V = \frac{1}{3}Bh$ *Use $\frac{1}{3}$ for k.*

$24\pi = \frac{1}{3}B(9)$ *Substitute.*

$8\pi = B$ *Solve for B.*

The base is 8π ft².

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Check It Out! Example 3

The lateral surface area L of a cone varies jointly as the area of the base radius r and the slant height l, and $L = 63\pi$ m² when $r = 3.5$ m and $l = 18$ m. Find r to the nearest tenth when $L = 8\pi$ m² and $l = 5$ m.

$L = krl$

$k = \pi$

$\boxed{r = 1.6\text{ m}}$

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A third type of variation describes a situation in which one quantity increases and the other decreases. For example, the table shows that the time needed to drive 600 miles decreases as speed increases.

Speed (mi/h)	Time (h)	Distance (mi)
30	20	600
40	15	600
50	12	600

This type of variation is an inverse variation. An **inverse variation** is a relationship between two variables x and y that can be written in the form $y = \frac{k}{x}$, where $k \neq 0$. For the equation $y = \frac{k}{x}$, y varies inversely as x.

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Example 4: Writing and Graphing Inverse Variation

$y = \frac{k}{x}$

Given: *y* varies inversely as *x*, and *y* = 4 when *x* = 5. Write the inverse variation function.

$y = \frac{k}{x}$ — *y varies inversely as x.*

$4 = \frac{k}{5}$ — *Substitute 4 for y and 5 for x.*

$k = 20$ — *Solve for k.*

$y = \frac{20}{x}$ — *Write the variation formula.*

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Check It Out! Example 4

Given: *y* varies inversely as *x*, and *y* = 4 when *x* = 10. Write the inverse variation function.

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When you want to find specific values in an inverse variation problem, you can solve for *k* and then use substitution or you can use the equation derived below.

$$y_1 = \frac{k}{x_1} \rightarrow y_1x_1 = k \quad \text{and} \quad y_2 = \frac{k}{x_2} \rightarrow y_2x_2 = k \quad \text{so,} \quad y_1x_1 = y_2x_2.$$

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Example 5: Sports Application

$t = \frac{k}{s}$

The time *t* needed to complete a certain race varies inversely as the runner's average speed *s*. If a runner with an average speed of 8.82 mi/h completes the race in 2.97 h, what is the average speed of a runner who completes the race in 3.5 h?

Method 1 Find *k*.

$t = \frac{k}{s}$

$2.97 = \frac{k}{8.82}$ — *Substitute.*

$k = 26.1954$ — *Solve for k.*

$t = \frac{26.1954}{s}$ — *Use 26.1954 for k.*

$3.5 = \frac{26.1954}{s}$ — *Substitute 3.5 for t.*

$s \approx 7.48$ — *Solve for s.*

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Example 5 Continued

Method Use $t_1s_1 = t_2s_2$.

$$t_1s_1 = t_2s_2$$

$$(2.97)(8.82) = 3.5s \quad \textit{Substitute.}$$

$$26.1954 = 3.5s \quad \textit{Simplify.}$$

$$7.48 \approx s \quad \textit{Solve for s.}$$

So the average speed of a runner who completes the race in 3.5 h is approximately 7.48 mi/h.

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Check It Out! Example 5

The time *t* that it takes for a group of volunteers to construct a house varies inversely as the number of volunteers *v*. If 20 volunteers can build a house in 62.5 working hours, how many working hours would it take 15 volunteers to build a house?

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You can use algebra to rewrite variation functions in terms of *k*.

Direct Variation

$$y = kx \rightarrow k = \underbrace{\frac{y}{x}}_{\text{Constant ratio}}$$

Inverse Variation

$$y = \frac{k}{x} \rightarrow k = \underbrace{xy}_{\text{Constant product}}$$

Notice that in direct variation, the *ratio* of the two quantities is constant. In inverse variation, the *product* of the two quantities is constant.

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Example 6: Identifying Direct and Inverse Variation

Determine whether each data set represents a direct variation, an inverse variation, or neither.

A.

x	6.5	13	104
y	8	4	0.5

In each case $xy = 52$. The product is constant, so this represents an inverse variation.

B.

x	5	8	12
y	30	48	72

In each case $\frac{y}{x} = 6$. The ratio is constant, so this represents a direct variation.

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Example 6: Identifying Direct and Inverse Variation

Determine whether each data set represents a direct variation, an inverse variation, or neither.

C.

x	3	6	8
y	5	14	21

$\frac{5}{3} \neq \frac{14}{6}$

Since xy and $\frac{y}{x}$ are not constant, this is neither a direct variation nor an inverse variation.

neither

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Check It Out! Example 6

Determine whether each data set represents a direct variation, an inverse variation, or neither.

6a.

x	3.75	15	5
y	12	3	9

6b.

x	1	40	26
y	0.2	8	5.2

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A **combined variation** is a relationship that contains both direct and inverse variation. Quantities that vary directly appear in the numerator, and quantities that vary inversely appear in the denominator.

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Example 7: Chemistry Application

The change in temperature of an aluminum wire varies inversely as its mass *m* and directly as the amount of heat energy *E* transferred. The temperature of an aluminum wire with a mass of 0.1 kg rises 5°C when 450 joules (J) of heat energy are transferred to it. How much heat energy must be transferred to an aluminum wire with a mass of 0.2 kg raise its temperature 20°C?

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Example 7 Continued

Step 1 Find *k*.

$\Delta T = \frac{kE}{m}$ *Combined variation*

$5 = \frac{k(450)}{0.1}$ *Substitute.*

$\frac{1}{900} = k$ *Solve for k.*

Step 2 Use the variation function.

$\Delta T = \frac{E}{900m}$ *Use $\frac{1}{900}$ for k.*

$20 = \frac{E}{900(0.2)}$ *Substitute.*

$3600 = E$ *Solve for E.*

The amount of heat energy that must be transferred is 3600 joules (J).

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direct: $y = kx$
inversely: $y = \frac{k}{x}$

Check It Out! Example 7

$V = \frac{kT}{P}$

The volume *V* of a gas varies inversely as the pressure *P* and directly as the temperature *T*. A certain gas has a volume of 10 liters (L), a temperature of 300 kelvins (K), and a pressure of 1.5 atmospheres (atm). If the gas is heated to 400K, and has a pressure of 1 atm, what is its volume?

$V = 10$ L
$T = 300$ K
$P = 1.5$ atm

$10 = \frac{k(300)}{1.5}$

$15 = k(300)$

$k = 0.05$

$V = 0.05(400)$

$V = 20$ L