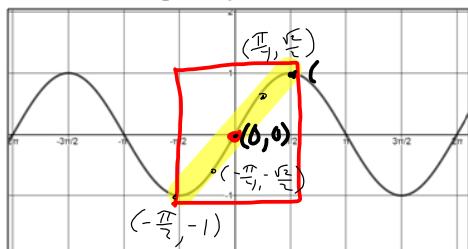


4.7 Inverse Trigonometric Functions

*Note: For a function to have an inverse function, it must be one-to-one; that is, it must pass the horizontal line test.

Graph of $y = \sin x$



Restrict the domain to:

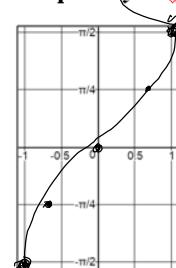
$$[-\frac{\pi}{2}, \frac{\pi}{2}]$$

On this interval:

- Sine is increasing
- range is $[-1, 1]$
- $y = \sin x$ is one-to-one
-

| | | | | | |
|--------------|------------------|-----------------------|---|----------------------|-----------------|
| $x = \sin y$ | -1 | $-\frac{\sqrt{2}}{2}$ | 0 | $\frac{\sqrt{2}}{2}$ | 1 |
| y | $-\frac{\pi}{2}$ | $-\frac{\pi}{4}$ | 0 | $\frac{\pi}{4}$ | $\frac{\pi}{2}$ |

Graph of $y = \arcsin x$

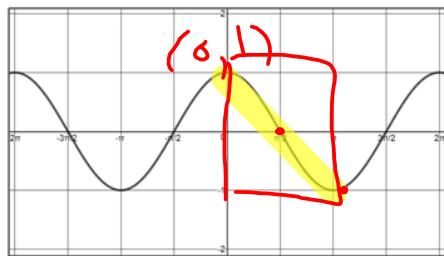


$$y = \sin^{-1} x$$

Domain:

$$[-1, 1]$$

Range: $[-\frac{\pi}{2}, \frac{\pi}{2}]$

Graph of $y = \cos x$ 

Restrict the domain to:

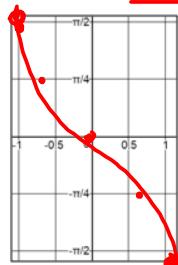
$$[0, \pi]$$

On this interval:

- cosine is decreasing
- range is $[-1, 1]$
- $y = \cos x$ is one-to-one

Graph of $y = \arccos x$

| | | | | | |
|--------------|---|----------------------|-----------------|-----------------------|-------|
| $x = \cos y$ | 1 | $\frac{\sqrt{2}}{2}$ | 0 | $-\frac{\sqrt{2}}{2}$ | -1 |
| y | 0 | $\frac{\pi}{4}$ | $\frac{\pi}{2}$ | $\frac{3\pi}{4}$ | π |

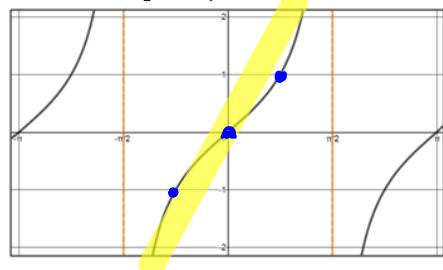


Domain:

$$[-1, 1]$$

Range:

$$[0, \pi]$$

Graph of $y = \tan x$ 

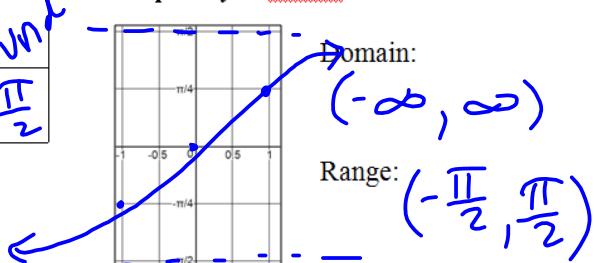
Restrict the domain to:

$$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

On this interval:

- tangent is increasing
- range is $(-\infty, \infty)$
- $y = \tan x$ is one-to-one

| $x = \tan y$ | undefined | -1 | 0 | 1 | undefined |
|--------------|--------------------|------------------|-----|-----------------|--------------------|
| y | $-\frac{\pi}{2}$ | $-\frac{\pi}{4}$ | 0 | $\frac{\pi}{4}$ | $\frac{\pi}{2}$ |

Graph of $y = \arctan x$ 

Summarize the domain and range for each inverse trigonometric function.

| Function | Domain | Range | Quadrants |
|-----------------|---------------------|-----------------------------------|----------------|
| $y = \arcsin x$ | $[-1, 1]$ | $[-\frac{\pi}{2}, \frac{\pi}{2}]$ | I, II |
| $y = \arccos x$ | $[-1, 1]$ | $[0, \pi]$ | I, II |
| $y = \arctan x$ | $(-\infty, \infty)$ | $(-\frac{\pi}{2}, \frac{\pi}{2})$ | I, II |



In 1 – 9, find the exact value of each expression without a calculator.

$$1. \arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$$

$$2. \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$$

$$3. \sin^{-1}(2) \text{ not possible}$$

$$4. \arccos\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$$

$$5. \cos^{-1}(-1) = \pi$$

$$6. \arccos\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$$

$$7. \arctan 0 = 0$$

$$8. \tan^{-1}(-1) = -\frac{\pi}{4}$$

$$9. \arctan\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$$

For You ☺ In 10 - 18, find the exact value of each expression without a calculator.

$$10. \arcsin(-1) = -\frac{\pi}{2}$$

$$11. \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

$$12. \sin^{-1}(\sqrt{3}) = \text{not possible}$$

$$13. \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$$

$$14. \arccos\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$$

$$15. \cos^{-1}(\sqrt{3}) = \text{not possible}$$

$$16. \arctan 1 = \frac{\pi}{4}$$

$$17. \tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) = -\frac{\pi}{6}$$

$$18. \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$$

In 19 - 30, find the exact value of each expression without a calculator.

19. $\tan(\arctan(-5)) = -5$

20. $\arcsin\left(\sin\frac{5\pi}{3}\right) = -\frac{\pi}{3}$
 $\arcsin\left(-\frac{\sqrt{3}}{2}\right)$

21. $\cos(\cos^{-1}1) = 1$

22. $\tan(\arctan(-14)) = -14$

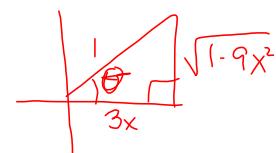
23. $\sin(\arcsin \pi)$

24. $\cos(\arccos(0.54))$

draw a \triangle to illustrate
 25. $\tan(\arccos\frac{2}{3}) = \frac{\sqrt{5}}{2}$

26. $\cos(\arcsin(-\frac{3}{5}))$

27. $\sin(\arccos 3x) = \sqrt{1-9x^2}$



$$(3x)^2 + b^2 = 1$$

$$9x^2 + b^2 = 1$$

$$b^2 = 1 - 9x^2$$

$$b = \sqrt{1-9x^2}$$

28. $\cot(\arccos 3x)$

29. $\cot\left(\arctan\frac{1}{x}\right)$

30. $\csc\left(\arctan\frac{x}{\sqrt{2}}\right)$