4-6 The Natural Base, *e*

Warm Up
Simplify.

1. $\log 10^x$ — x

2. $\log_b b^{3w}$ — $3w$

3. $10^{\log z}$ — z

4. $b^{\log_b (x-1)}$ — $x - 1$

5. $\left(\frac{1}{3}\right)3^{(x-1)}$ — 3^{x-2}

4-6 The Natural Base, *e*

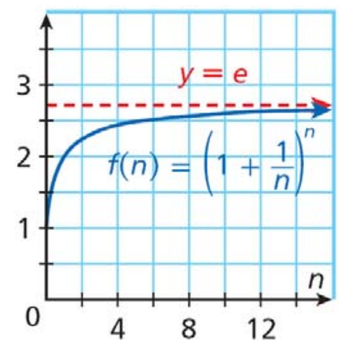
Recall the *compound interest formula* $A = P(1 + \frac{r}{n})^{nt}$, where A is the amount, P is the principal, r is the annual interest, n is the number of times the interest is compounded per year and t is the time in years.

Suppose that \$1 is invested at 100% interest ($r = 1$) compounded n times for one year as represented by the function $f(n) = P(1 + \frac{1}{n})^n$.

4-6 The Natural Base, *e*

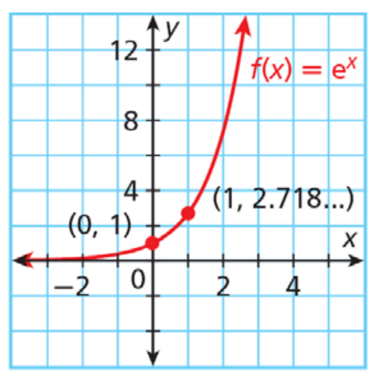
As *n* gets very large, interest is *continuously compounded*. Examine the graph of $f(n) = (1 + \frac{1}{n})^n$. The function has a horizontal asymptote. As *n* becomes infinitely large, the value of the function approaches approximately 2.7182818…. This number is called *e*. Like π, the constant *e* is an irrational number.

e



4-6 The Natural Base, *e*

Exponential functions with *e* as a base have the same properties as the functions you have studied. The graph of $f(x) = e^x$ is like other graphs of exponential functions, such as $f(x) = 3^x$.



The domain of $f(x) = e^x$ is all real numbers. The range is $\{y | y > 0\}$.

D: $(-\infty, \infty)$

R: $(0, \infty)$ HA: $y = 0$

4-6 The Natural Base, *e*

Caution

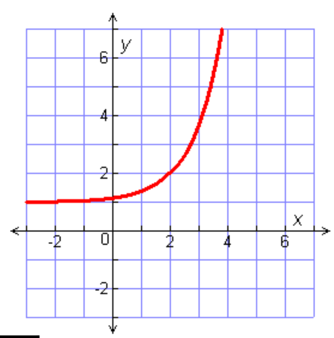
The decimal value of e looks like it repeats: 2.718281828… The value is actually 2.71828182890… There is no repeating portion.

4-6 The Natural Base, *e*

Example 1: Graphing Exponential Functions

Graph $f(x) = e^{x-2} + 1$.

Make a table. Because *e* is irrational, the table values are rounded to the nearest tenth.



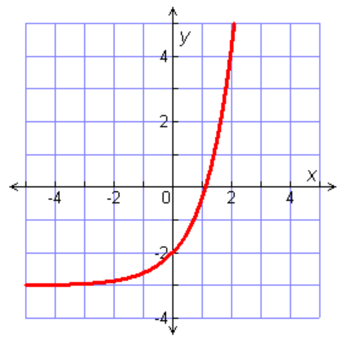
x	–2	–1	0	1	2	3	4
$f(x) = e^{x-2} + 1$	1.0	1.0	1.1	1.4	2	3.7	8.4

4-6 The Natural Base, *e*

Check It Out! Example 1

Graph $f(x) = e^x - 3$.

Make a table. Because *e* is irrational, the table values are rounded to the nearest tenth.



x	–4	–3	–2	–1	0	1	2
$f(x) = e^x - 3$	–3	–3	–2.9	–2.7	–2	–0.3	4.4

4-6 The Natural Base, *e*

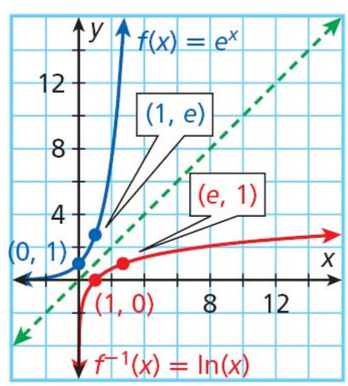
ln means $\log_e$

A logarithm with a base of *e* is called a **natural logarithm** and is abbreviated as "ln" (rather than as $\log_e$). Natural logarithms have the same properties as log base 10 and logarithms with other bases.

ln

The **natural logarithmic function** $f(x) = \ln x$ is the inverse of the natural exponential function $f(x) = e^x$.

D: $(0, \infty)$
R: $(-\infty, \infty)$
VA: $x = 0$

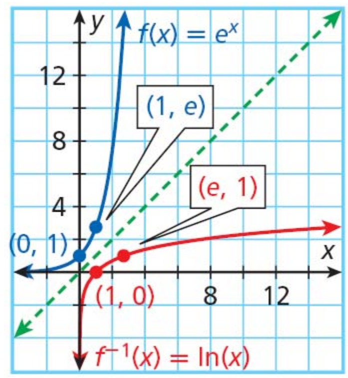


4-6 The Natural Base, *e*

The domain of $f(x) = \ln x$ is $\{x|x > 0\}$.

The range of $f(x) = \ln x$ is all real numbers.

All of the properties of logarithms from Lesson 4-3 also apply to natural logarithms.



4-6 The Natural Base, *e*

Example 2: Simplifying Expression with *e* or ln

Simplify.

A. $\ln e^{0.15t}$

$\log_e e^{0.15t} = 0.15t$

B. $e^{3\ln(x+1)}$

$e^{\ln(x+1)^3} = (x+1)^3$

C. $\ln_e e^{2x} + \ln_e e^{x}$

$2x + x$

$3x$

OR

$\ln(e^{2x} \cdot e^{x})$

$\ln_e e^{3x} = 3x$

4-6 The Natural Base, *e*

Check It Out! Example 2

Simplify.

a. $\ln e^{3.2}$

$\ln e^{3.2} = 3.2$

b. $e^{2\ln x}$

$e^{2\ln x} = x^2$

c. $\ln e^{x+4y}$

$\ln e^{x+4y} = x + 4y$

4-6 The Natural Base, *e*

The formula for continuously compounded interest is $A = Pe^{rt}$, where A is the total amount, P is the principal, r is the annual interest rate, and t is the time in years.

4-6 The Natural Base, *e*

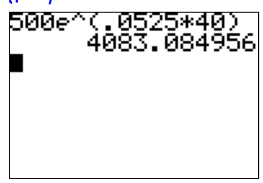
Example 3: Economics Application

What is the total amount for an investment of $500 invested at 5.25% for 40 years and compounded continuously?

$A = Pe^{rt}$

$A = 500e^{0.0525(40)}$ *Substitute 500 for P, 0.0525 for r, and 40 for t.*

$A \approx 4083.08$ *Use the e^x key on a calculator.*



The total amount is $4083.08.

4-6 The Natural Base, *e*

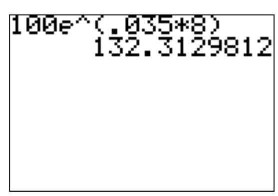
Check It Out! Example 3

What is the total amount for an investment of $100 invested at 3.5% for 8 years and compounded continuously?

$A = Pe^{rt}$

$A = 100e^{0.035(8)}$ *Substitute 100 for P, 0.035 for r, and 8 for t.*

$A \approx 132.31$ *Use the e^x key on a calculator.*



The total amount is $132.31.

4-6 The Natural Base, *e*

The *half-life* of a substance is the time it takes for half of the substance to breakdown or convert to another substance during the process of decay. Natural decay is modeled by the function below.

$A = Pe^{rt}$ exp growth

N_0 is the initial amount (at $t = 0$). *k* is the decay constant.

$$N(t) = N_0 e^{-kt}$$ exp decay

$N(t)$ is the amount remaining. *t* is the time.

4-6 The Natural Base, *e*

Example 4: Science Application

$N(t) = N_0 e^{-kt}$

Pluonium-239 (Pu-239) has a half-life of 24,110 years. How long does it take for a 1 g sample of Pu-239 to decay to 0.1 g?

$N_0 = 1g$, [When $t = 24{,}110$ yrs, $N(t) = 0.5g$]

Find k

$0.5 = 1e^{-k(24{,}110)}$

$0.5 = e^{-24110k}$

$\ln 0.5 = \ln e^{-24110k}$

$\ln 0.5 = -24110k$

$k = \frac{\ln 0.5}{-24110} =$

$k = 0.0000287$

4-6 The Natural Base, *e*

$N(t) = N_0 e^{-kt}$

$0.1 = 1e^{-0.0000287t}$

$0.1 = e^{-0.0000287t}$

$\ln 0.1 = \ln e^{-0.0000287t}$

$\frac{\ln 0.1}{-0.0000287} = \frac{-0.0000287t}{-0.0000287}$

$t = 80229.45$ years

Holt McDougal Algebra 2

4-6 The Natural Base, *e*

Check It Out! Example 4

Determine how long it will take for 650 mg of a sample of chromium-51 which has a half-life of about 28 days to decay to 200 mg.

Step 1 Find the decay constant for Chromium-51.

$N(t) = N_0e^{-kt}$	*Use the natural decay function. t.*
$\frac{1}{2} = 1e^{-k(28)}$	*Substitute 1 for N_0,28 for t, and $\frac{1}{2}$ for N(t) because half of the initial quantity will remain.*

Holt McDougal Algebra 2

4-6 The Natural Base, *e*

Check It Out! Example 4 Continued

$\ln \frac{1}{2} = \ln e^{-28k}$ — *Simplify and take ln of both sides.*

$\ln 2^{-1} = -28k$ — *Write $\frac{1}{2}$ as 2^{-1}, and simplify the right side.*

$-\ln 2 = -28k$ — *$\ln 2^{-1} = -1\ln 2 = -\ln 2$.*

$$k = \frac{\ln 2}{28} \approx 0.0247$$

4-6 The Natural Base, *e*

Check It Out! Example 4 Continued

Step 2 Write the decay function and solve for *t*.

$N(t) = N_0e^{-0.0247t}$ — *Substitute 0.0247 for k.*

$200 = 650e^{-0.0247t}$ — *Substitute 650 for N_0 and 200 for N(t).*

$\ln \frac{200}{650} = \ln e^{-0.0247t}$ — *Take ln of both sides.*

$\ln \frac{200}{650} = -0.0247t$ — *Simplify.*

$$t = \frac{\ln \frac{200}{650}}{-0.0247} \approx 47.7$$

It takes approximately 47.7 days to decay.