4-5 Exponential and Logarithmic Equations and Inequalities

An **exponential equation** is an equation containing one or more expressions that have a variable as an exponent. To solve exponential equations:

- Try writing them so that the bases are all the same.
- Take the logarithm of both sides.

If $b^x = b^y$, then $x = y$ $(b \neq 0, b \neq 1)$.

If $a = b$, then $\log a = \log b$ $(a > 0, b > 0)$.

4-5 Exponential and Logarithmic Equations and Inequalities

Example 1A: Solving Exponential Equations

Solve and check.

$9^{8-x} = 27^{x-3}$

$(3^2)^{8-x} = (3^3)^{x-3}$

$3^{16-2x} = 3^{3x-9}$

$16 - 2x = 3x - 9$

$-5x = -25$

$x = 5$

Holt McDougal Algebra 2
Copyright © by Holt Mc Dougal. All Rights Reserved.

Example 1B: Solving Exponential Equations

Solve and check.

$4^{x-1} = 5$ $\quad b^x = a$

$\log_4 5 = x - 1$

$\log_4 4^{x-1} = \log_4 5$

$x - 1 = \log_4 5$

$x = \log_4 5 + 1 \rightarrow \frac{\log 5}{\log 4} + 1$

$x = 2.161$

4-5 Exponential and Logarithmic Equations and Inequalities

Check It Out! Example 1a

Solve and check.

1A $3^{2x} = 27$

$x = 3/2$

1B $7^{-x} = 21$

$x = -1.565$

1C $2^{3x} = 15$

$x = 1.302$

Check It Out! Example 1b

Solve and check.

$7^{-x} = 21$

4-5 Exponential and Logarithmic Equations and Inequalities

Check It Out! Example 1c

Solve and check.

$2^{3x} = 15$

$\frac{\log_2 15}{3} = \frac{3x}{3}$

$x = \frac{\log_2 15}{3} = \left(\frac{\log 15}{\log 2}\right) \div 3$

$x = 1.302$

$2^{3x} = 15$

$\log 2^{3x} = \log 15$

$\frac{3x \cdot \log 2}{\log 2} = \frac{\log 15}{\log 2}$

$x = \left(\frac{\log 15}{\log 2}\right) \div 3$

4-5 Exponential and Logarithmic Equations and Inequalities

A **logarithmic equation** is an equation with a logarithmic expression that contains a variable. To solve logartihmic equations:

- Rewrite the logarithmic equation in exponential form; solve.
- Use the properties of logarithms. Think:
  If $\log_b x = \log_b y$ then $x = y$

CHECK FOR EXTRANEOUS SOLUTIONS!!

4-5 Exponential and Logarithmic Equations and Inequalities

Example 2A: Solving Logarithmic Equations

Solve.

$\log_6(2x - 1) = -1$

Check
$\log_6 \frac{1}{6} = -1$ ✓

$6^{-1} = 2x - 1$

$\frac{1}{6} = 2x - 1$

$2x = \frac{7}{6}$

$x = \frac{7}{12}$

Example 2B: Solving Logarithmic Equations

Solve.

$\log_4 100 - \log_4(x + 1) = 1$

$\log_4 \frac{100}{x+1} = 1$

$4 = \frac{100}{x+1}$

$x + 1 = 25$

$x = 24$

Example 2C: Solving Logarithmic Equations

Solve.

$\log_5 x^4 = 8$

$\pm\sqrt[4]{5^8} = \sqrt[4]{x^4}$

$\pm 5^2 = x$

$x = \pm 25$