4-5 Exponential and Logarithmic Equations and Inequalities

An **exponential equation** is an equation containing one or more expressions that have a variable as an exponent. To solve exponential equations:

- Try writing them so that the bases are all the same.

  If $b^x = b^y$, then $x = y$ $(b \neq 0, b \neq 1)$.

- Take the logarithm of both sides.

  If $a = b$, then $\log a = \log b$ $(a > 0, b > 0)$.

4-5 Exponential and Logarithmic Equations and Inequalities

Example 1A: Solving Exponential Equations

Solve and check.

$$9^{8-x} = 27^{x-3}$$

$$\left(3^2\right)^{8 \cdot x} = \left(3^3\right)^{x-3}$$

$$3^{16-2x} = 3^{3x-9}$$

$$16 - 2x = 3x - 9$$

$$-5x = -25$$

$$\boxed{x = 5}$$

4-5 Exponential and Logarithmic Equations and Inequalities

Example 1B: Solving Exponential Equations

Solve and check.

$4^{x-1} = 5$

$$\log 4^{x-1} = \log 5$$

$$(x-1)\log 4 = \frac{\log 5}{\log 4}$$

$$x - 1 = \frac{\log 5}{\log 4}$$

$$X = 1 + \frac{\log 5}{\log 4}$$

$$\boxed{X = 2.161}$$

$$4^{x-1} = 5$$

$$\log_4 4^{x-1} = \log_4 5$$

$$x - 1 = \log_4 5$$

$$X = \log_4 5 + 1$$

$$X = \frac{\log 5}{\log 4} + 1$$

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4-5 Exponential and Logarithmic Equations and Inequalities

Check It Out! Example 1a

Solve and check.

$3^{2x} = 27$

$$3^{2x} = 3^3$$

$$2x = 3$$

$$x = 3/2$$

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4-5 Exponential and Logarithmic Equations and Inequalities

Check It Out! Example 1b

Solve and check.

$7^{-x} = 21$

$$\log_7 7^{-x} = \log_7 21$$

$$\frac{-x}{-1} = \frac{\log_7 21}{-1}$$

$$x = -\log_7 21 = -\frac{\log 21}{\log 7} \approx \boxed{-1.565}$$

4-5 Exponential and Logarithmic Equations and Inequalities

Check It Out! Example 1c

Solve and check.

$2^{3x} = 15$

4-5 **Exponential and Logarithmic Equations and Inequalities**

A **logarithmic equation** is an equation with a logarithmic expression that contains a variable. To solve logartihmic equations:

- Rewrite the logarithmic equation in exponential form; solve.

- Use the properties of logarithms. Think:
  If $\log_b x = \log_b y$ then $x = y$

CHECK FOR EXTRANEOUS SOLUTIONS!!

4-5 **Exponential and Logarithmic Equations and Inequalities**

Example 2A: Solving Logarithmic Equations

Solve.

$\log_6(2x - 1) = -1$

$$6^{-1} = 2x - 1$$

$$\frac{1}{6} = 2x - 1$$

$$\frac{2x}{2} = \frac{\frac{7}{6}}{2} \qquad \boxed{x = \frac{7}{12}}$$

4-5 **Exponential and Logarithmic Equations and Inequalities**

Example 2B: Solving Logarithmic Equations

Solve.

$$\log_4 100 - \log_4(x + 1) = 1$$

$$\log_4\left(\frac{100}{x+1}\right) = 1$$

$$4^1 = \frac{100}{x+1}$$

$$4 = \frac{100}{x+1}$$

$$x + 1 = 25$$

$$\boxed{x = 24}$$

4-5 **Exponential and Logarithmic Equations and Inequalities**

Example 2C: Solving Logarithmic Equations

Solve.

$$\log_5 x^4 = 8$$

$$\pm\sqrt[4]{5^8} = \sqrt[4]{x^4}$$

$$x = \pm 5^2$$

$$\boxed{x = \pm 25}$$

4-5 Exponential and Logarithmic Equations and Inequalities

Example 2D: Solving Logarithmic Equations

Solve.

2D

$\log_{12} x + \log_{12}(x + 1) = 1$

$\log_{12} x(x+1) = 1$

$\log_{12} x^2 + x = 1$

$\rightarrow x^2 + x = 12$

$x^2 + x - 12 = 0$

$(x + 4)(x - 3) = 0$

$x = -4, \boxed{3}$

2E $\quad 2\log x - \log 4 = 0$

$\log\left(\frac{x^2}{4}\right) = 0$

$\frac{x^2}{4} = 10^0$

$\frac{x^2}{4} = 1$

$x^2 = 4$

$x = \pm 2 \boxed{x=2}$