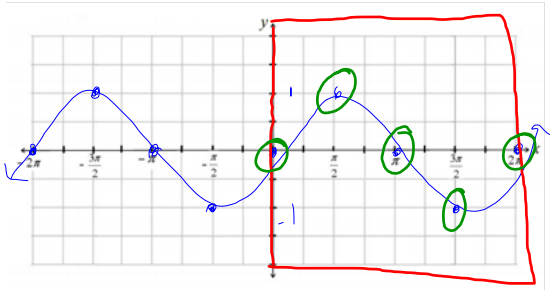
4.5 Graphs of Sine and Cosine Functions

The Sine Curve

x	$y = \sin x$
-2π	0
$-\frac{3\pi}{2}$	1
$-\pi$	0
$-\frac{\pi}{2}$	-1
0	0
$\frac{\pi}{2}$	1
π	0
$\frac{3\pi}{2}$	-1
2π	0



The Sine Curve

The Basics

x **Domain:** $(-\infty, \infty)$ All real #'s

y **Range:** $[-1, 1]$

Period: 2π

Even/Odd: $f(-x) = -f(x)$

symmetric about the origin

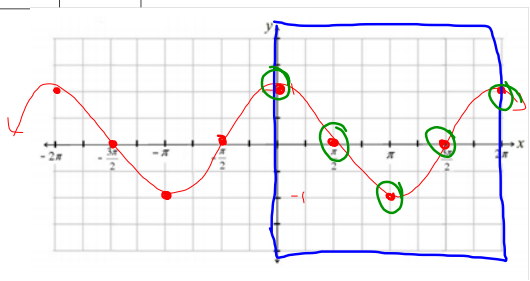
Asymptotes: none

Key Points:

zero/intercept, max, zero/int, min, zero/int.

The Cosine Curve

x	$y = \cos x$
-2π	1
$-\frac{3\pi}{2}$	0
$-\pi$	-1
$-\frac{\pi}{2}$	0
0	1
$\frac{\pi}{2}$	0
π	-1
$\frac{3\pi}{2}$	0
2π	1



The Cosine Curve

The Basics

Domain: $(-\infty, \infty)$

Range: $[-1, 1]$

Period: 2π

Even/Odd: $f(-x) = f(x)$

symmetric about the y-axis

Asymptotes: none

Key Points:

max, zero/int., min, zero/int, max

Transformations of Sine and Cosine Curves

or cos

Standard Form: $f(x) = a\sin(bx - c) + d$

Amplitude: $|a|$ height of curve (vertical stretch/shrink)

Reflection: reflected in x-axis $a < 0$

(maxs become mins, mins become maxs)

Period: $\frac{2\pi}{|b|}$ $|b| < 1$ horizontal stretch by $\frac{1}{b}$ (per. $> 2\pi$)

$|b| > 1$ hor. shrink by $\frac{1}{b}$ (per $< 2\pi$)

Phase Shift (AKA Horizontal Translation): $\frac{c}{b}$

Vertical Translation: d

Midline: "line of oscillation"

$y = d$

Range: $d \pm |a|$

Additional Notes:

- all graphs should include at least 2 full periods
- identify transformations before labeling your axes
- "key point" intervals $\frac{\text{period}}{4}$

use this to scale x-axis

In 1 – 3, identify the amplitude and period.

$b = \frac{\pi}{3}$

1. $y = \frac{1}{4}\sin 2\pi x$

$\frac{2\pi}{2\pi}$

amp $= \frac{1}{4}$

period: 1

2. $y = 2\cos 3x$

amp $= 2$

period $= \frac{2\pi}{3}$

3. $y = \frac{1}{2}\sin\frac{\pi x}{3}$

amp $= \frac{1}{2}$

per $= 6$

$\frac{2\pi}{\pi/3} = \frac{2\cancel{\pi}}{1} \cdot \frac{3}{\cancel{\pi}}$

In 4 – 7, describe the relationship between the graphs of *g* and *f*.

4. $f(x) = \sin x; g(x) = \sin(x - \pi)$

horizontal

g(x) is a phase shift to the right π

5. $f(x) = \cos 2x; g(x) = -\cos 2x$

g(x) is a reflection across x-axis

6. $f(x) = \cos x; g(x) = \cos 2x$

g(x) is horizontal compression of f(x), by 1/2

OR

period of g(x) is π (vs. period of f(x) is 2π)

7. $f(x) = \sin x; g(x) = \sin x + 3$

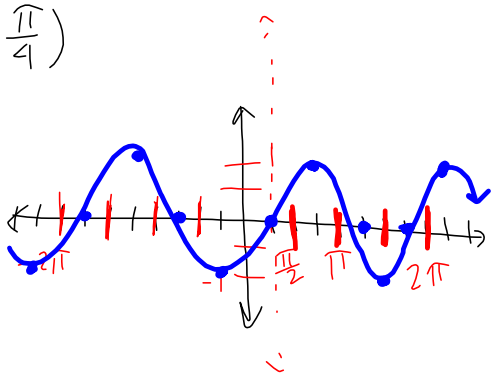
g(x) is a vertical shift up 3

45. $y = \sin(x - \frac{\pi}{4})$

amp = 1

per = 2π

shift right $\frac{\pi}{4}$



In 8 – 11, sketch the graph of each function. Identify any and all transformations of the parent graph first, and be sure to include two full periods.

8. $y = 2\sin 2x$

Amplitude: 2

Reflection: no

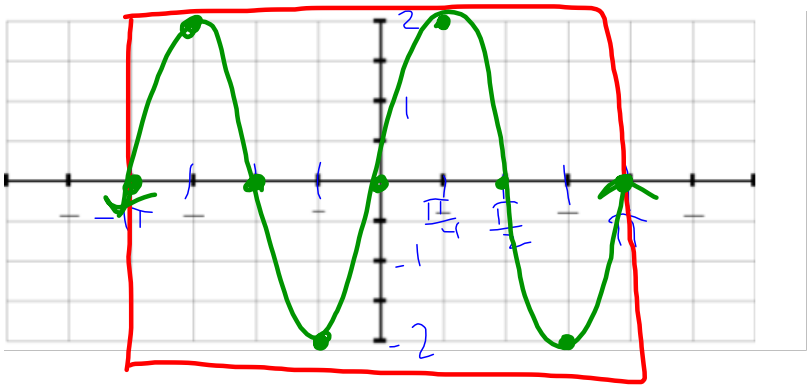
Period: $\frac{2\pi}{2} = \pi$

Phase Shift: no

Vertical Translation: no

Midline: $y = 0$

Range: $[-2, 2]$



x-axis scale: $\frac{\text{period}}{4} = \frac{\pi}{4}$

9. $y = 2\cos x - 3$

Amplitude: 2

Reflection: no

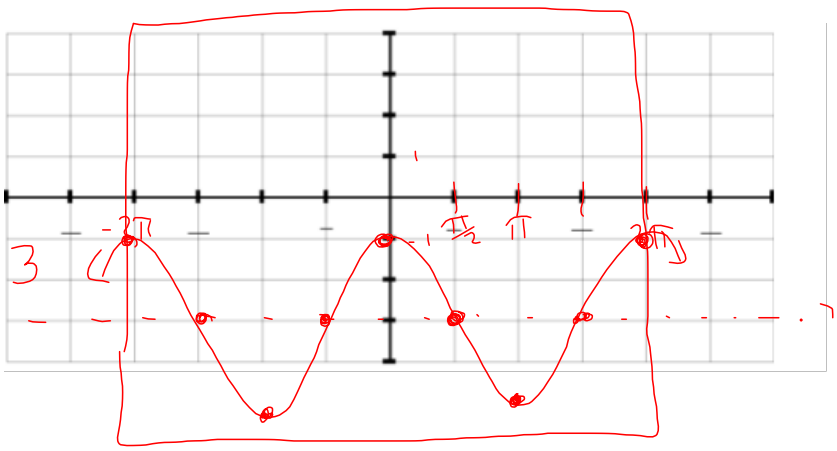
Period: 2π

Phase Shift: none

Vertical Translation: down 3

Midline: $y = -3$

Range: $[-5, -1]$



x-axis scale: $\frac{2\pi}{4} = \frac{\pi}{2}$

10. $y = \frac{1}{2}\sin(x - \pi)$

Amplitude: $\frac{1}{2}$

Reflection: no

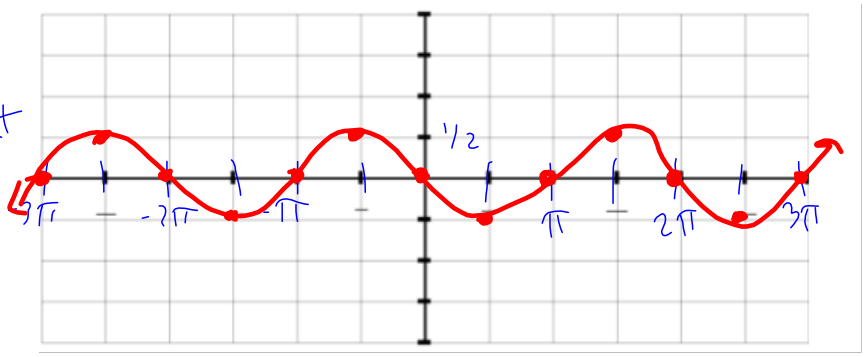
Period: 2π

Phase Shift: $\frac{c}{b} = \frac{\pi}{1}$ right

Vertical Translation: none

Midline: $y = 0$

Range: $[-\frac{1}{2}, \frac{1}{2}]$



11. $y = -\cos\left(x + \frac{\pi}{2}\right)$

Amplitude: 1

Reflection: yes

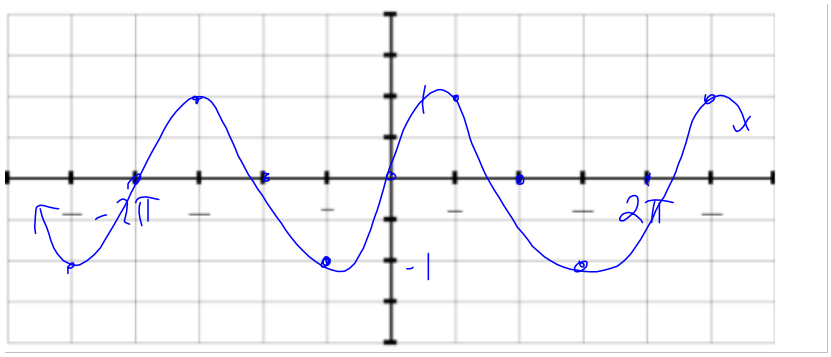
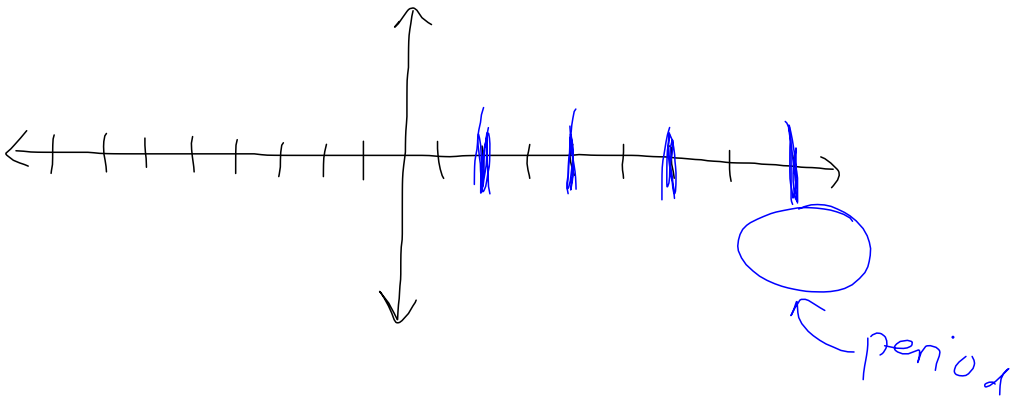
Period: 2π

Phase Shift: left $\frac{\pi}{2}$

Vertical Translation: no

Midline: $y = 0$

Range: $[-1, 1]$



Graphing Template