4.4 Factoring Polynomials

Core Concept

The Factor Theorem

A polynomial $f(x)$ has a factor $x - k$ if and only if $f(k) = 0$.

Determine whether

(a) $x - 2$ is a factor of $f(x) = x^2 + 2x - 4$

$$\begin{array}{r|rrr} 2 & 1 & 2 & -4 \\ & & 2 & 8 \\ \hline & 1 & 4 & |4 \end{array}$$

NO → b/c $f(2) \neq 0$

Because remainder does not $= 0$.

(b) $x + 5$ is a factor of $f(x) = 3x^4 + 15x^3 - x^2 + 25$.

$$\begin{array}{r|rrrrr} -5 & 3 & 15 & -1 & 0 & 25 \\ & & -15 & 0 & 5 & -25 \\ \hline & 3 & 0 & -1 & 5 & |0 \end{array}$$

Yes

$f(-5) = 0$

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$$-1 \,\big|\, 1 \quad -3 \quad 1$$
$$\quad\quad\quad -1 \quad 4$$
$$\overline{\quad 1 \quad -4 \,\big|\, 5}$$

NO

$P(-1) \neq 0$

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$$-2 \,\big|\, 3 \quad 6 \quad 0 \quad -5 \quad -10$$
$$\quad\quad\quad -6 \quad 0 \quad 0 \quad 10$$
$$\overline{\quad 3 \quad 0 \quad 0 \quad -5 \,\big|\, 0}$$

yes

$(x-2) = 0$

Factoring, revisited.

What we already know..........

Factor each polynomial completely.

a. $x^3 - 4x^2 - 5x$ **b.** $3y^5 - 48y^3$ **c.** $5z^4 + 30z^3 + 45z^2$

$x(x^2 - 4x - 5)$ $3y^3(y^2 - 16)$ $5z^2(z^2 + 6z + 9)$

$\boxed{x(x-5)(x+1)}$ $\boxed{3y^3(y+4)(y-4)}$ $\boxed{5z^2(z+3)^2}$

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Check it Out!

Factor the polynomial completely.

1. $x^3 - 7x^2 + 10x$

$x(x^2 - 7x + 10)$

$\boxed{x(x-2)(x-5)}$

2. $3n^7 - 75n^5$

$3n^5(n^2 - 25) = 3n^5(n+5)(n-5)$

3. $8m^5 - 16m^4 + 8m^3 = 8m^3(m^2 - 2m + 1)$

$\boxed{8m^3(m-1)(m-1)}$ $8m^3(m-1)^2$

FACTOR

$x^3 + x^2 + 16x + 16$

$x^2(x+1) + 16(x+1)$

$\boxed{(x+1)(x^2+16)}$

FACTOR

$4x^3 - 8x^2 - x + 2$

$4x^2(x-2) - 1(x-2)$

$(x-2)(4x^2-1)$ $(4x^2-1)(x-2)$

$\boxed{(x-2)(2x+1)(2x-1)}$

Factoring by Grouping   Used for polynomials with four or more terms.

Factor: $x^3 - x^2 - 25x + 25.$

$x^2(x-1) - 25(x-1)$

$(x-1)(x^2-25)$

$(x-1)(x+5)(x-5)$

Check it out!

Factor: $x^3 - 2x^2 - 9x + 18.$

$x^2(x-2) - 9(x-2)$

$(x-2)(x^2-9)$

$(x-2)(x+3)(x-3)$

Core Concept

Perfect Cubes

$1, 8, 27, 64, 125, 216, \ldots$

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Special Factoring Patterns

Sum of Two Cubes

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Example

$64x^3 + 1 = (4x)^3 + 1^3$

$= (4x + 1)(16x^2 - 4x + 1)$

Difference of Two Cubes
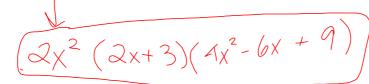
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Example

$27x^3 - 8 = (3x)^3 - 2^3$

$= (3x - 2)(9x^2 + 6x + 4)$

Factor completely.

$a^3 - b^3$

(a) $x^3 - 125$

$(a - b)(a^2 + ab + b^2)$

$a : x$

$b : 5$

$(x - 5)(x^2 + 5x + 25)$

(b) ~~16x^5 + 54x^2~~ $16x^5 + 54x^2$

$2x^2(8x^3 + 27)$

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$a : 2x \qquad (2x)^3 = 8x^3$

$b : 3$

$2x^2(2x + 3)(4x^2 - 6x + 9)$

Factor completely.

(a) $z^6 + 8 = (z^2+2)(z^4-2z^2+4)$

$a: z^2$
$b: 2$

$a: x \quad b: 2$

(b) $2x^5 - 16x^2 = 2x^2(x^3-8)$
$\boxed{2x^2(x-2)(x^2+2x+4)}$

Factoring Polynomials in Quadratic Form

$a(\quad)^2 + b(\quad) + c$

Example 5a
Factor $\underline{x^4} + 2\underline{x^2} - 63$ completely. $(x^2)^2 + 2(x^2)-63$

$(x^2+9)(x^2-7)$

Example 5b

Factor $3x^4 + 22x^2 + 40$ completely. $\quad$ Think $\quad 3a^2 + 22a + 40$

$(3x^2+10)(x^2+4)$

$3x^4 + 10x^2 + 12x^2 + 40$

Factor

$(x)^2 + 2(x) - 63$

$(x + 9)(x - 7)$

Example 5c

Factor $3p^8 + 15p^5 + 18p^2$ **completely.**

$3p^2(p^6 + 5p^3 + 6)$

$3p^2(p^3 + 3)(p^3 + 2)$

Factor $16x^4 - 81$ completely. $a^2 - b^2 = (a+b)(a-b)$

$(4x^2 + 9)(4x^2 - 9)$

$(4x^2 + 9)(2x + 3)(2x - 3)$

Check it Out! Examples

Factor each expression completely.

5a. $3x^4 - 3x^2 - 168$ **5b.** $x^4 - 3x^2 - 10$

5c. $4x^4 + 44x^2y^2 + 112y^4$ **5d.** $49x^4 - 1$

Additional Practice. Factor *completely*.

1. $a^3 + 27$ **2.** $6z^5 - 750z^2$

3. $x^3 + 4x^2 - x - 4$ **4.** $3y^3 + y^2 + 9y + 3$

5. $-16n^4 + 625$ **6.** $5w^6 - 25w^4 + 30w^2$

Show that $x + 3$ is a factor of $f(x) = x^4 + 3x^3 - x - 3$. Then factor $f(x)$ completely.

During the first 5 seconds of a roller coaster ride, the function $h(t) = 4t^3 - 21t^2 + 9t + 34$ represents the height h (in feet) of the roller coaster after t seconds. How long is the roller coaster at or below ground level in the first 5 seconds?

$h(t) = 4t^3 - 21t^2 + 9t + 34$