4.4 Factoring Polynomials

Core Concept

The Factor Theorem

A polynomial $f(x)$ has a factor $x - k$ if and only if $f(k) = 0$.

Determine whether

(a) $x - 2$ is a factor of $f(x) = x^2 + 2x - 4$

2 | 1 2 -4
 2 8
1 4 | 4

NO → b/c $f(2) \neq 0$

Because remainder does not = 0.

(b) $x + 5$ is a factor of $f(x) = 3x^4 + 15x^3 - x^2 + 25$.

-5 | 3 15 -1 0 25
 -15 0 5 -25
3 0 -1 5 | 0

Yes $f(-5) = 0$

(A)

$$\begin{array}{r|rrr} -1 & 1 & -3 & 1 \\ & & -1 & 4 \\ \hline & 1 & -4 & 5 \end{array}$$

NO

$P(-1) \neq 0$

(B)

$$\begin{array}{r|rrrrr} -2 & 3 & 6 & 0 & -5 & -10 \\ & & -6 & 0 & 0 & 10 \\ \hline & 3 & 0 & 0 & -5 & 0 \end{array}$$

yes

$P(-2) = 0$

Factoring, revisited.

What we already know..........

Factor each polynomial completely.

a. $x^3 - 4x^2 - 5x$

$x(x^2 - 4x - 5)$

$x(x-5)(x+1)$

b. $3y^5 - 48y^3$

$3y^3(y^2 - 16)$

$3y^3(y+4)(y-4)$

c. $5z^4 + 30z^3 + 45z^2$

$5z^2(z^2 + 6z + 9)$

$5z^2(z+3)^2$

Factoring by Grouping

Used for polynomials with four or more terms.

Factor: $x^3 - x^2 - 25x + 25.$

$x^2(x-1) - 25(x-1)$

$(x-1)(x^2-25)$

$(x-1)(x+5)(x-5)$

Check it out!

Factor: $x^3 - 2x^2 - 9x + 18.$

$x^2(x-2) - 9(x-2)$

$(x-2)(x^2-9)$

$(x-2)(x+3)(x-3)$