Remember that to *multiply* powers with the same base, you *add* exponents.

$$b^m b^n = b^{m+n}$$

Product Property of Logarithms

For any positive numbers m, n, and b $(b \neq 1)$,

WORDS	NUMBERS	ALGEBRA
The logarithm of a product is equal to the sum of the logarithms of its factors.	$\log_3 1000 = \log_3(10 \cdot 100)$ $= \log_3 10 + \log_3 100$	$\log_b mn = \log_b m + \log_b n$

Example 1

Express as a single logarithm. Simplify, if possible.

$\log_6 4 + \log_6 9 = \log_6 (4 \cdot 9) = \log_6 36 = 2$

$4 \quad + \quad 2$

$\log_5 625 + \log_5 25 = \log_5 15625 = 6$

$\log_{\frac{1}{3}} 27 + \log_{\frac{1}{3}} \frac{1}{9} = \log_{1/3} 3 = -1$

Remember that to *divide* powers with the same base, you *subtract* exponents

$$\frac{b^m}{b^n} = b^{m-n}$$

Because logarithms are exponents, subtracting logarithms with the same base is the same as finding the logarithms of the quotient with that base.

Quotient Property of Logarithms

For any positive numbers m, n, and b $(b \neq 1)$,

WORDS	NUMBERS	ALGEBRA
The logarithm of a quotient is the logarithm of the dividend minus the logarithm of the divisor.	$\log_5\left(\frac{16}{2}\right) = \log_5 16 - \log_5 2$	$\log_b \frac{m}{n} = \log_b m - \log_b n$

Example 2: Subtracting Logarithms

Express $\log_5 100 - \log_5 4$ as a single logarithm. Simplify, if possible.

$$\log_5\left(\frac{100}{4}\right) = \log_5 25 = 2$$

Check It Out! Example 2

Express $\log_7 49 - \log_7 7$ as a single logarithm. Simplify, if possible.

$\log_7 7 = 1$

$(a^m)^n$

Because you can multiply logarithms, you can also take powers of logarithms.

Power Property of Logarithms

For any real number p and positive numbers a and b $(b \neq 1)$,

WORDS	NUMBERS	ALGEBRA
The logarithm of a power is the product of the exponent and the logarithm of the base.	$\log 10^3$ $\log(10 \cdot 10 \cdot 10)$ $\log 10 + \log 10 + \log 10$ $3 \log 10$	$\boxed{\log_b a^p = p \log_b a}$

Example 3: Simplifying Logarithms with Exponents

Express as a product. Simplify, if possible.

A. $\log_2 32^6$

$6 \log_2 32$

$6(5) = 30$

B. $\log_8 4^{20}$

$20 \boxed{\log_8 4}$

$8^{2/3} = 4$

$(\sqrt[3]{8})^2 =$

$20\left(2/3\right) = 40/3$

Check It Out! Example 3

Express as a product. Simplify, if possibly.

a. $\log_{10} 10^4$ $= 4$

$4 \log_{10} 10$

b. $\log_5 25^2$ $= 4$

$2 \log_5 25$

Exponential and logarithmic operations undo each other since they are inverse operations.

Inverse Properties of Logarithms and Exponents

For any base b such that $b > 0$ and $b \neq 1$,

ALGEBRA	EXAMPLE
$\log_b b^x = x$	$\log_{10} 10^7 = 7$
$b^{\log_b x} = x$	$10^{\log_{10} 2} = 2$

Example 4: Recognizing Inverses

Simplify each expression.

a. $\log_3 3^{11}$

11

b. $\log_3 81$

$\log_3 3^4 = 4$

c. $5^{\log_5 10} = 10$

Check It Out! Example 4

a. Simplify $\log 10^{0.9}$

$\log 10^{0.9}$ 9

0.9

b. Simplify $2^{\log_2(8x)}$

$2^{\log_2(8x)}$ 8x

$8x$

Most calculators calculate logarithms only in base 10 or base *e*. You can change a logarithm in one base to a logarithm in another base with the following formula.

Change of Base Formula

For $a > 0$ and $a \neq 1$ and any base b such that $b > 0$ and $b \neq 1$,

ALGEBRA	EXAMPLE
$\log_b x = \dfrac{\log_a x}{\log_a b}$	$\log_4 8 = \dfrac{\log_2 8}{\log_2 4}$

Example 5: Changing the Base of a Logarithm

Evaluate $\log_{32} 8$. $= \frac{3}{5}$

$$\frac{\log_2 8}{\log_2 32} = \frac{3}{5}$$

$$\frac{\log 8}{\log 32} = 0.6$$

Check It Out! Example 5a

Evaluate $\log_9 27$.

$$\frac{\log_3 27}{\log_3 9} = \frac{3}{2}$$

Check It Out! Example 5b

Evaluate $\log_8 16$.

$$\frac{\log_2 16}{\log_2 8} = \frac{4}{3}$$

$$\log_b x = \frac{\log_a x}{\log_a b}$$

19

$\log_3 5^2 = 2.92$

$2\boxed{\log_3 5}$

$2\left(\frac{\log 5}{\log 3}\right) = 2(1.46) = 2.92$