

B. Synthetic Division

This method can only be used when dividing by a linear binomial whose leading coefficient is 1. In other words, this method can only be used when dividing by a polynomial of the form $(x - k)$.

Let's see how it works.

Divide the following using synthetic division.

$(x^3 + 2x^2 - 6x - 9) \div (x + 3) = x^2 - x - 3$

(x - k)

k (1) $(x^3 + 2x^2 - 6x - 9) \div (x + 3) = x^2 - x - 3$

-3	1	2	-6	-9	← coefficients of dividend
	-3	-3	3	9	
multiply	-3	-3	3	9	
	1	-1	-3	0	← remainder
		↑	↑	↑	
		coefficients	of	answer	

MUST insert placeholder

Divide the following using synthetic division.

(2) $(x^3 - 7x - 6) \div (x - 2) = x^2 + 2x - 3 - 12/x - 2$

0x²

2	1	0	-7	-6
	↓	2	4	-6
	1	2	-3	-12

(3) $(4x^2 + 5x - 4) \div (x + 1)$

-1	4	5	-4	$4x + 1 - 5/x + 1$
	-4	-4	-1	
	4	1	-5	

Remainder Theorem

If the polynomial function $P(x)$ is divided by $x - a$, then the remainder r is $P(a)$.

Example:

If your last name begins with A - L, find the value of $P(3)$ if

$$P(x) = x^3 - 4x^2 + 5x + 1.$$

$$P(3) = 3^3 - 4(3)^2 + 5(3) + 1 = 7$$

If your last name begins with M - Z, use synthetic division to divide $x^3 - 4x^2 + 5x + 1$ by $x - 3$.

$$\begin{array}{r|rrrr} 3 & 1 & -4 & 5 & 1 \\ & & 3 & -3 & 6 \\ \hline & 1 & -1 & 2 & 7 \end{array}$$

Use synthetic substitution to evaluate the polynomial for the given value.

(1) $P(x) = x^3 - 4x^2 + 3x - 5$ for $x = 4$

$$P(4) = 7$$

$$\begin{array}{r|rrrr} 4 & 1 & -4 & 3 & -5 \\ & \downarrow & 4 & 0 & 12 \\ \hline & 1 & 0 & 3 & 7 \end{array}$$

(2) $P(x) = 4x^4 + 2x^3 + 3x + 5$ for $x = -\frac{1}{2}$

$$P(-\frac{1}{2}) = \frac{7}{2}$$

$$\begin{array}{r|rrrrr} -\frac{1}{2} & 4 & 2 & 0 & 3 & 5 \\ & \downarrow & -2 & 0 & 0 & -\frac{3}{2} \\ \hline & 4 & 0 & 0 & 3 & \frac{7}{2} \end{array}$$