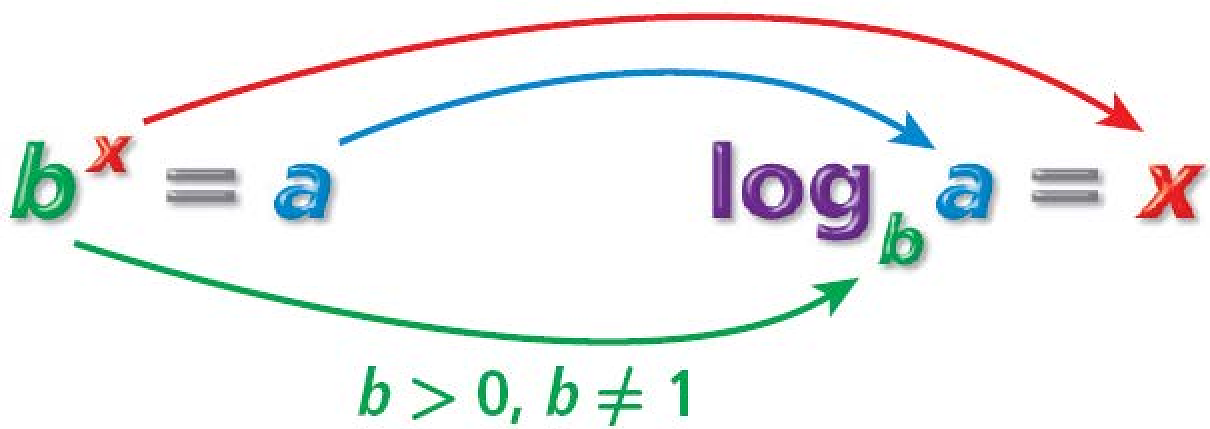
You can write an exponential equation as a logarithmic equation and vice versa.



Reading Math

Read $\log_b a = x$, as "the log base b of a is x." Notice that the log is the exponent.

Example 1: Converting from Exponential to Logarithmic Form

Write each exponential equation in logarithmic form.

Exponential Equation	Logarithmic Form
$3^5 = 243$	$\log_3 243 = 5$
$25^{\frac{1}{2}} = 5$	$\log_{25} 5 = \frac{1}{2}$
$10^4 = 10{,}000$	$\log_{10} 10000 = 4$
$6^{-1} = \frac{1}{6}$	$\log_6 \frac{1}{6} = -1$
$a^b = c$	$\log_a c = b$

Check It Out! Example 1

Write each exponential equation in logarithmic form.

	Exponential Equation	Logarithmic Form
a.	$9^2 = 81$	$\log_9 81 = 2$
b.	$3^3 = 27$	$\log_3 27 = 3$
c.	$x^0 = 1 (x \neq 0)$	$\log_x 1 = 0$

Example 2: Converting from Logarithmic to Exponential Form

Write each logarithmic equation in exponential form.

Logarithmic Form	Exponential Equation
$\log_9 9 = 1$	$9^1 = 9$
$\log_2 512 = 9$	$2^9 = 512$
$\log_8 2 = \frac{1}{3}$	$8^{1/3} = 2$
$\log_4 \frac{1}{16} = -2$	$4^{-2} = \frac{1}{16}$
$\log_b 1 = 0$	$b^0 = 1$

Check It Out! Example 2

Write each logarithmic equation in exponential form.

Logarithmic Form	Exponential Equation
$\log_{10}10 = 1$	$10^1 = 10$
$\log_{12}144 = 2$	$12^2 = 144$
$\log_{\frac{1}{2}}8 = -3$	$\frac{1}{2}^{-3} = 8$

A logarithm is an exponent, so the rules for exponents also apply to logarithms. You may have noticed the following properties in the last example.

Special Properties of Logarithms

For any base b such that $b > 0$ and $b \neq 1$,

LOGARITHMIC FORM	EXPONENTIAL FORM	EXAMPLE
Logarithm of Base b		
$\log_b b = 1$	$b^1 = b$	$\log_{10} 10 = 1$ $10^1 = 10$
Logarithm of 1		
$\log_b 1 = 0$	$b^0 = 1$	$\log_{10} 1 = 0$ $10^0 = 1$

A logarithm with base 10 is called a **common logarithm**. If no base is written for a logarithm, the base is assumed to be 10. For example, $\log 5 = \log_{10} 5$.

You can use mental math to evaluate some logarithms.

Example 3A: Evaluating Logarithms by Using Mental Math

Evaluate by using mental math. No Calculators!

$\log_{10} 0.01 = \log_{10} \frac{1}{100} = -2$ b/c $10^{-2} = \frac{1}{100}$

$\log_5 125 = 3$

$\log_{125} 5 = 1/3$ $\quad 125^{1/3} = 5$

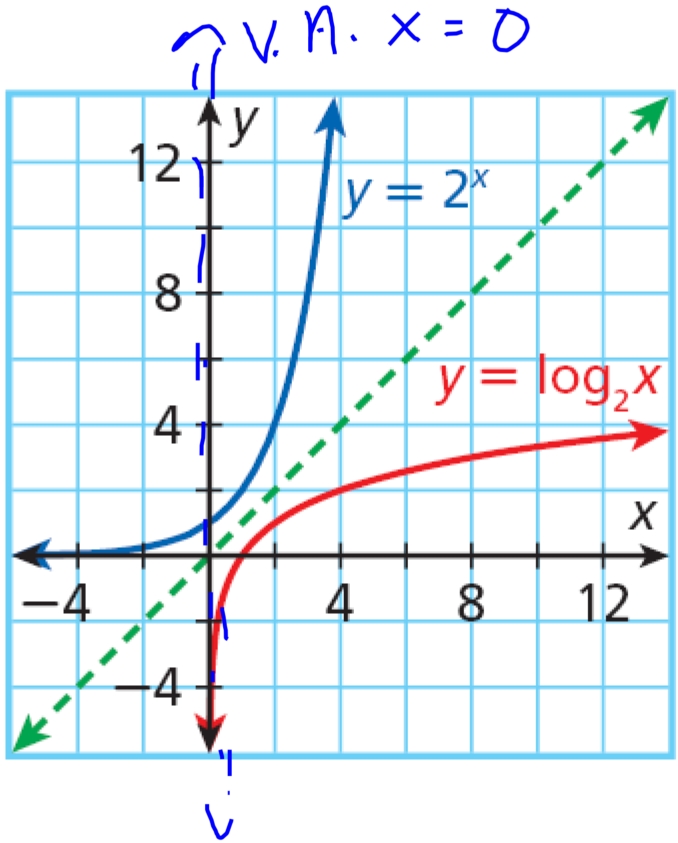
$\log_5 \frac{1}{5} = -1$

$\log 0.00001 = -5$ $\quad \log \frac{1}{100{,}000}$

$\log_{25} 0.04 = -1$ $\quad \log_{25} \frac{4}{100} = \log_{25} \frac{1}{25} = -1$

Because logarithms are the inverses of exponents, the inverse of an exponential function, such as $y = 2^x$, is a **logarithmic function**, such as $y = \log_2 x$.

You may notice that the domain and range of each function are switched.



The domain of $y = 2^x$ is all real numbers ($\mathbb{R}$), and the range is $\{y|y > 0\}$. The domain of $y = \log_2 x$ is $\{x|x > 0\}$, and the range is all real numbers ($\mathbb{R}$).

Example 4A: Graphing Logarithmic Functions

$f^{-1}(x) = \log_{1.25} x$

Use the *x*-values {–2, –1, 0, 1, 2}. Graph the function and its inverse. Describe the domain and range of the inverse function.

D: $(0, \infty)$
R: $(-\infty, \infty)$

$f(x) = ab^x$

$f(x) = (1.25^x)$ → HA: $y = 0$

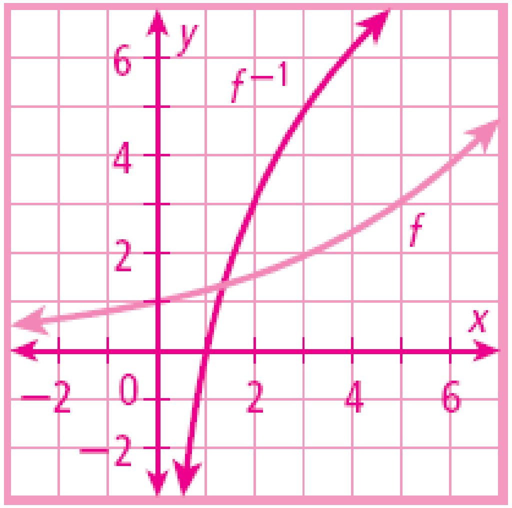
x	-2	-1	0	1	2
f(x)	0.64	0.8	1	1.25	1.5625

x	0.64	0.8	1	1.25	1.5625
$f^{-1}(x)$	-2	-1	0	1	2

VA: $x = 0$

Example 4A Continued

To graph the inverse, $f^{-1}(x) = \log_{1.25}x$, by using a table of values.



x	0.64	0.8	1	1.25	1.5625
$f^{-1}(x) = \log_{1.25}x$	–2	–1	0	1	2

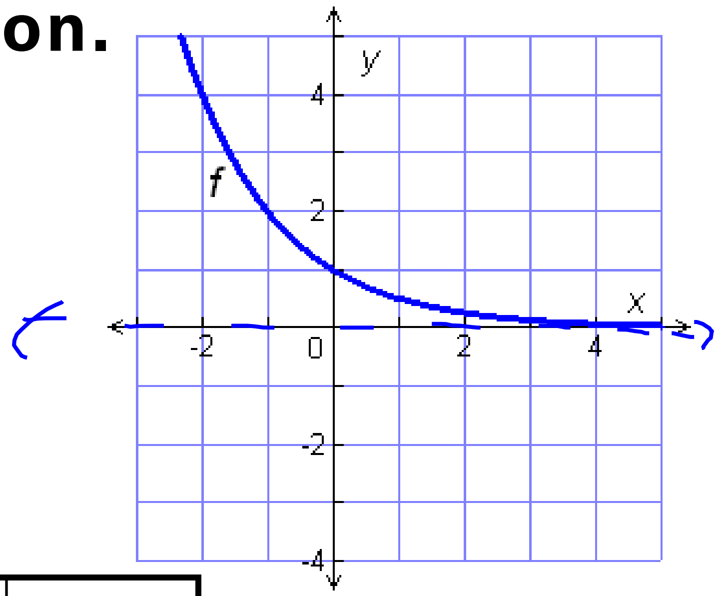
The domain of $f^{-1}(x)$ is $\{x|x > 0\}$, and the range is $\mathbb{R}$.

Example 4B: Graphing Logarithmic Functions

Use the *x*-values {–2, –1, 0, 1, 2}. Graph the function and its inverse. Describe the domain and range of the inverse function.

$$f(x) = \left(\frac{1}{2}\right)^x$$

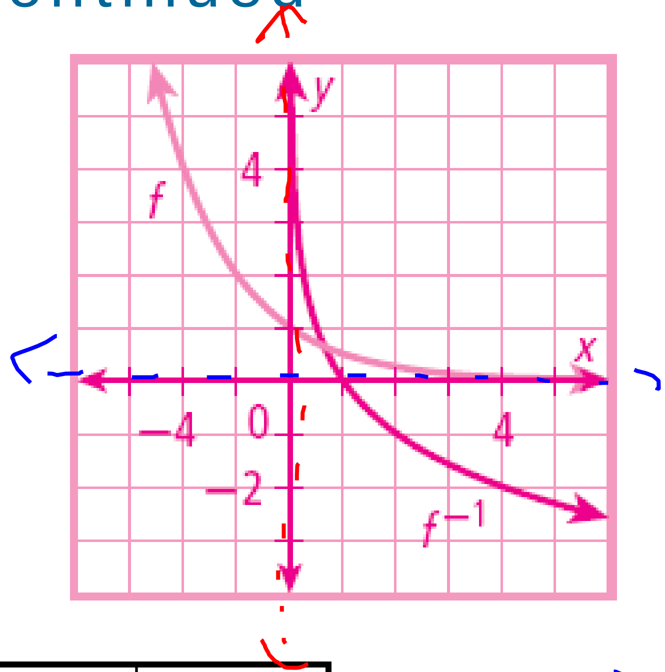
Graph $f(x) = \frac{1}{2}^x$ by using a table of values.



x	–2	–1	0	1	2
$f(x) = \left(\frac{1}{2}\right)^x$	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$

Example 4B Continued

To graph the inverse, $f^{-1}(x) = \log_{\frac{1}{2}} x$, by using a table of values.



x	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$
$f^{-1}(x) = \log_{\frac{1}{2}} x$	–2	–1	0	1	2

D: $(0, \infty)$
R: $(-\infty, \infty)$

The domain of $f^{-1}(x)$ is $\{x | x > 0\}$, and the range is ℝ.

Helpful Hint

The LOG key is used to evaluate logarithms in base 10. 2nd LOG (10^x) is used to find 10^x, the inverse of log.

Example 5: Food Application

The table lists the hydrogen ion concentrations for a number of food items. Find the pH of each.

Substance	H^+ conc. (mol/L)
Milk	0.00000025
Tomatoes	0.0000316
Lemon juice	0.0063

Example 5 Continued

Milk

The hydrogen ion concentration is 0.00000025 moles per liter.

$pH = -\log[H^+]$

$pH = -\log(0.00000025)$ *Substitute the known values in the function.*

Use a calculator to find the value of the logarithm in base 10. Press the LOG key.

```
-log(0.00000025)
        6.602059991
```

Milk has the pH of about 6.6.

Example 5 Continued

Tomatoes

The hydrogen ion concentration is 0.0000316 moles per liter.

$pH = -\log[H^+]$

$pH = -\log(0.0000316)$

Substitute the known values in the function.

Use a calculator to find the value of the logarithm in base 10. Press the LOG key.

```
-log(0.0000316)
     4.500312917
```

Tomatoes have the pH of about 4.5.

Example 5 Continued

Lemon juice

The hydrogen ion concentration is 0.0063 moles per liter.

$pH = -\log[H^+]$

$pH = -\log(0.0063)$

Substitute the known values in the function.

Use a calculator to find the value of the logarithm in base 10. Press the LOG key.

```
-log(0.0063)
     2.200659451
```

Lemon juice has the pH of about 2.2.