

4.3 Right Triangle Trigonometry

4.4 Trigonometric Functions of Any Angle

PART I

Right Triangle Definitions of Trigonometric Functions

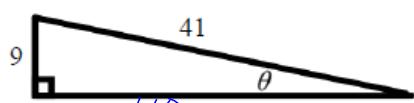
Let θ be an *acute* angle of a right triangle. The six trigonometric functions of the angle θ are defined as follows:

$$\begin{aligned}\sin \theta &= \frac{\text{opposite}}{\text{hypotenuse}} = \frac{o}{H} & \cos \theta &= \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{A}{H} & \tan \theta &= \frac{o}{A} \\ \csc \theta &= \frac{H}{o} & \sec \theta &= \frac{H}{A} & \cot \theta &= \frac{A}{o}\end{aligned}$$



Find the exact values of the six trigonometric functions of the angle shown in the figure.

1.



$$\begin{aligned}9^2 + b^2 &= 41^2 \\ b^2 &= 1600 \\ b &= 40\end{aligned}$$

$$\begin{aligned}\sin \theta &= \frac{9}{41} & \csc \theta &= \frac{41}{9} \\ \cos \theta &= \frac{40}{41} & \sec \theta &= \frac{41}{40} \\ \tan \theta &= \frac{9}{40} & \cot \theta &= \frac{40}{9}\end{aligned}$$

In 2 – 3, sketch a right triangle corresponding to the trigonometric function of the acute angle. Use the Pythagorean Theorem to determine the third side of the triangle and then find the other five trigonometric functions of the angle.

$$2. \sin \theta = \frac{3}{8} \quad \begin{array}{c} O \\ H \end{array}$$

$$\begin{aligned} 3^2 + b^2 &= 8^2 \\ 9 + b^2 &= 64 \\ b^2 &= 55 \end{aligned}$$

$$\csc \theta = \frac{8}{3} \quad 3. \csc \theta = \frac{17}{4}$$

$$\cos \theta = \frac{\sqrt{55}}{8} \quad \sec \theta = \frac{8}{\sqrt{55}} = \frac{8\sqrt{55}}{55}$$

$$\tan \theta = \frac{3}{\sqrt{55}} = \frac{3\sqrt{55}}{55} \quad \cot \theta = \frac{\sqrt{55}}{3}$$

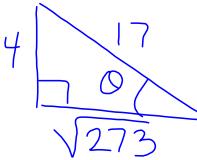
$$\sin \theta = \frac{3}{17}$$

$$\cos \theta = \frac{\sqrt{273}}{17}$$

$$\sec \theta = \frac{17\sqrt{273}}{273}$$

$$\tan \theta = \frac{4\sqrt{273}}{273}$$

$$\cot \theta = \frac{\sqrt{273}}{4}$$



Definitions of Trigonometric Functions of Any Angle

Let θ be an angle in standard position with (x, y) a point on the terminal side of θ and $r = \sqrt{x^2 + y^2} \neq 0$.

$$\sin \theta = \frac{y}{r}$$

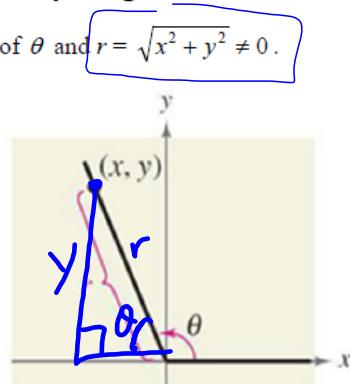
$$\cos \theta = \frac{x}{r}$$

$$\tan \theta = \frac{y}{x}$$

$$\csc \theta = \frac{r}{y}$$

$$\sec \theta = \frac{r}{x}$$

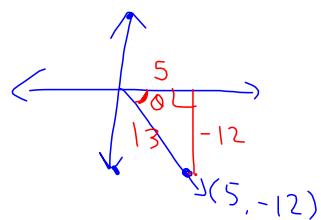
$$\cot \theta = \frac{x}{y}$$



In 4 – 5, the given point is on the terminal side of an angle in standard position. Determine the exact values of the six trigonometric functions of the angle.

4. $(7, 24)$

5. $(5, -12)$



$$\sin \theta = -\frac{12}{13} \quad \csc \theta = -\frac{13}{12}$$

$$\cos \theta = \frac{5}{13} \quad \sec \theta = \frac{13}{5}$$

$$\tan \theta = -\frac{12}{5} \quad \cot \theta = -\frac{5}{12}$$