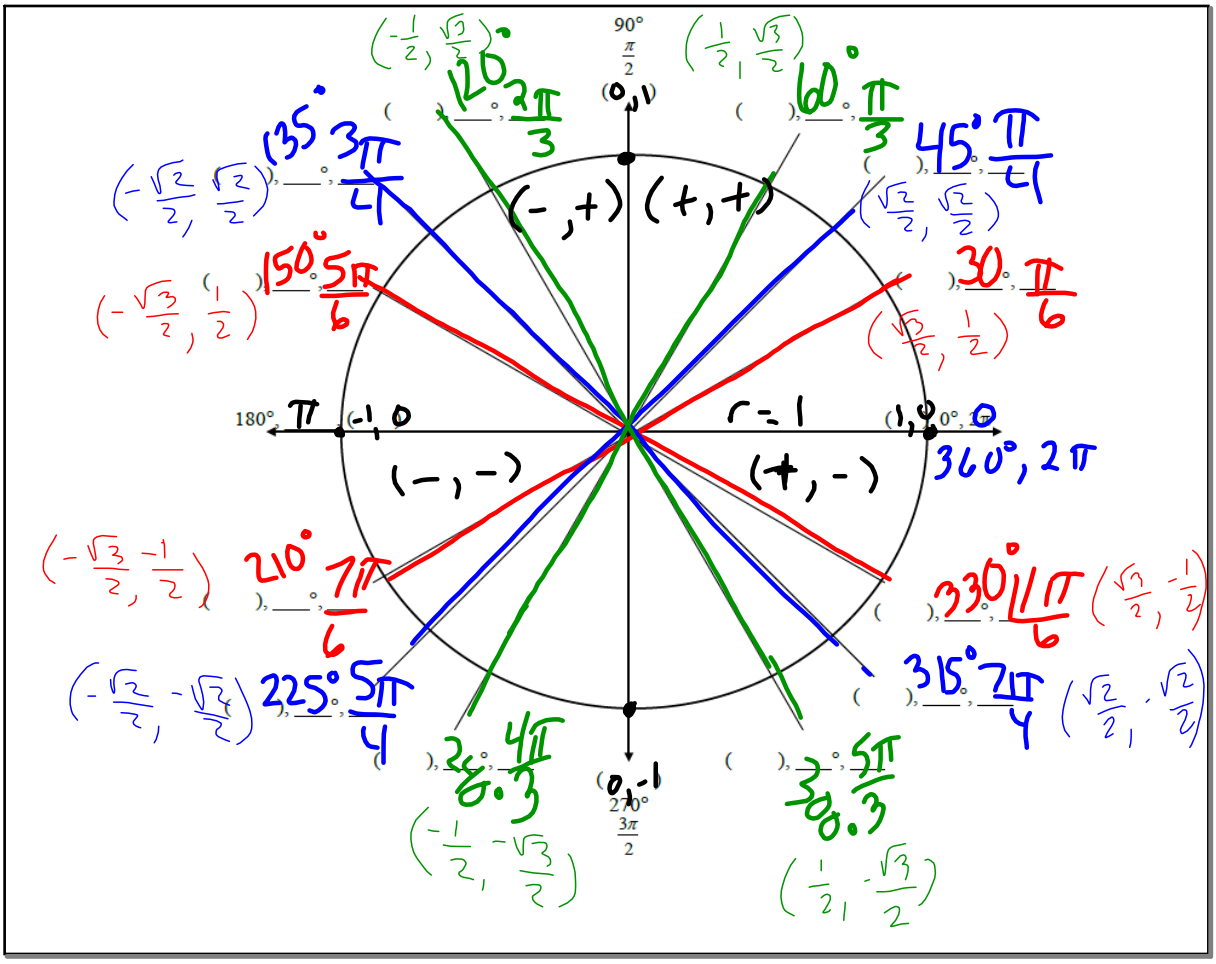


4.2 Trigonometric Functions: The Unit Circle

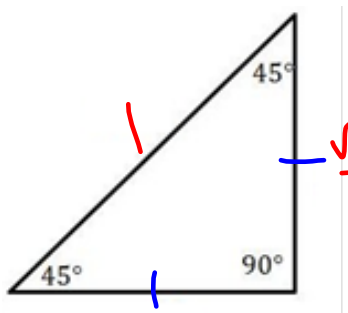
Geometry Quick Review: Special Right Triangles

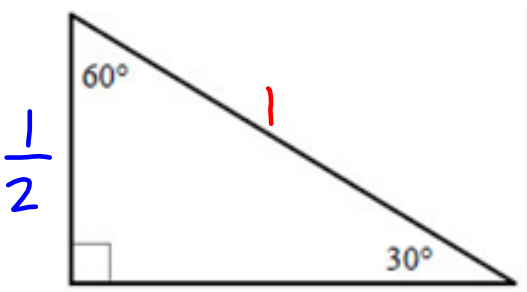
$1 : 1 : \sqrt{2}$

$x : x : x\sqrt{2}$

$1 : \sqrt{3} : 2$

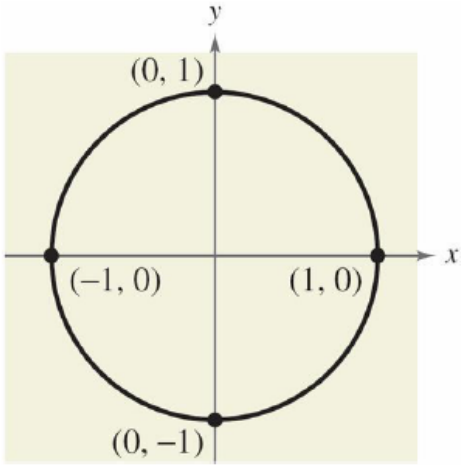
$a : a\sqrt{3} : 2a$



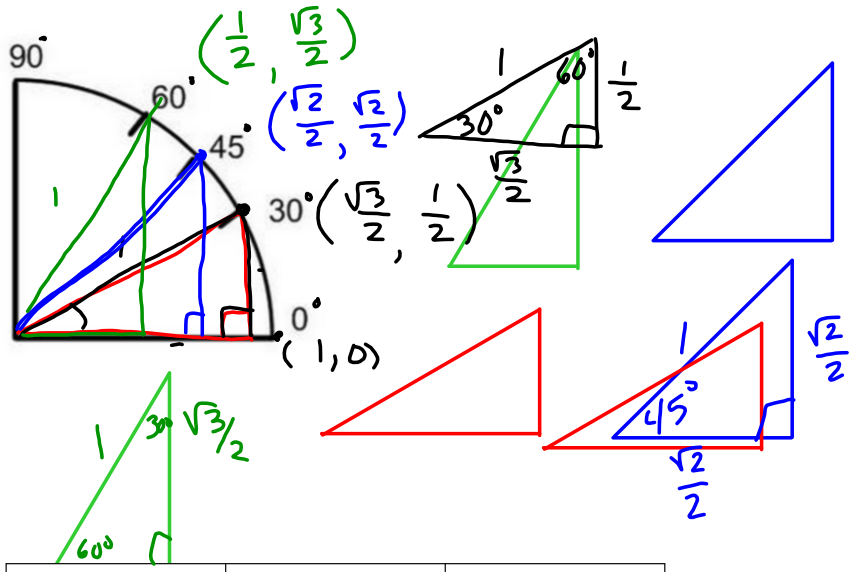


The Unit Circle: $x^2 + y^2 = 1$

↳ radius = 1

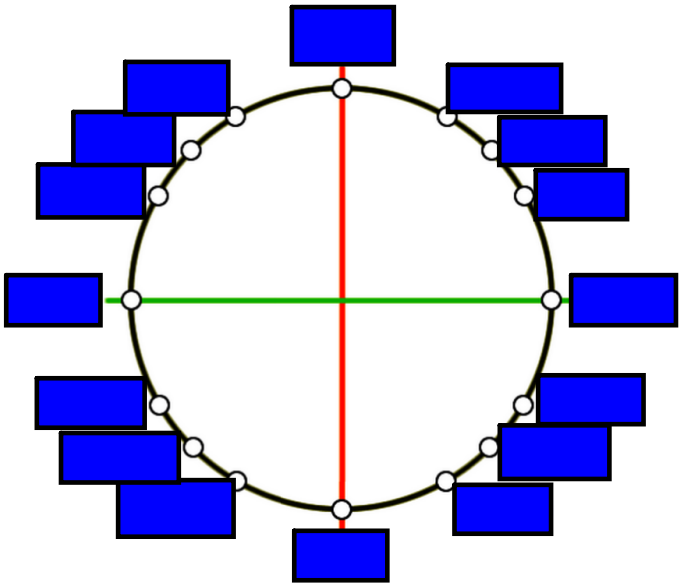


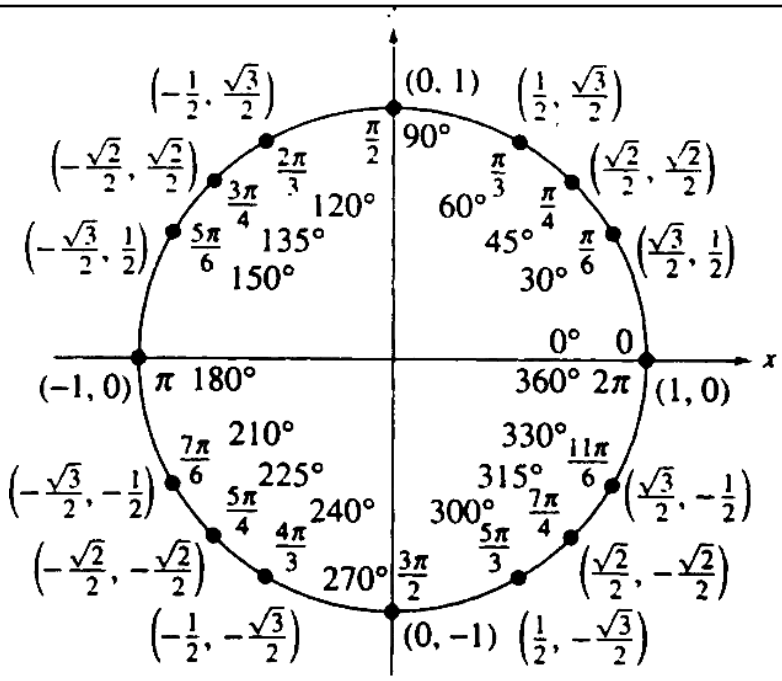
The Unit Circle: Quadrant I



Degree	Radian	Point (x, y)
$0°$	0	$(1, 0)$
$30°$	$\frac{\pi}{6}$	$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
$45°$	$\frac{\pi}{4}$	$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
$60°$	$\frac{\pi}{3}$	$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$90°$	$\frac{\pi}{2}$	$(0, 1)$

Ordered Pairs





The Trigonometric Functions

Let t be a real number and let (x, y) be the point on the unit circle corresponding to t.

Sine

$\sin t = y$

Cosine

$\cos t = x$

Tangent

$\tan t = \dfrac{\sin t}{\cos t} = \dfrac{y}{x}$

Cosecant

$\csc t = \dfrac{1}{\sin t} = \dfrac{1}{y};\ y \neq 0$

Secant

$\sec t = \dfrac{1}{\cos t} = \dfrac{1}{x};\ x \neq 0$

Cotangent

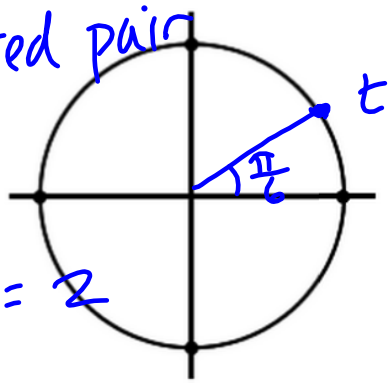
$\cot t = \dfrac{\cos t}{\sin t} = \dfrac{x}{y};\ y \neq 0$

Evaluate the six trigonometric functions at the given value.

1.) $t = \dfrac{\pi}{6}$

identify ordered pair

$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$



$\sin\frac{\pi}{6} = \frac{1}{2}$

$\csc\frac{\pi}{6} = 2$

$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$\sec\frac{\pi}{6} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\tan\frac{\pi}{6} = \frac{1/\cancel{2}}{\sqrt{3}/\cancel{2}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\hookrightarrow \frac{\sqrt{3}}{3}$

$\cot\frac{\pi}{6} = \frac{\sqrt{3}/\cancel{2}}{1/\cancel{2}} = \sqrt{3}$

Evaluate the six trigonometric functions at the given value.

2.) $t = \frac{5\pi}{4}$ $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

$\sin \frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$ $\csc \frac{5\pi}{4} = -\frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{2\sqrt{2}}{2} = -\sqrt{2}$

$\cos \frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$ $\sec \frac{5\pi}{4} = -\sqrt{2}$

$\tan \frac{5\pi}{4} = 1$ $\cot \frac{5\pi}{4} = 1$

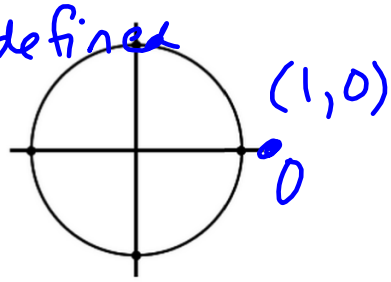
Evaluate the six trigonometric functions at the given value.

3.) $t = 0$

$\sin 0 = 0$ $\csc 0 = \frac{1}{0}$ undefined

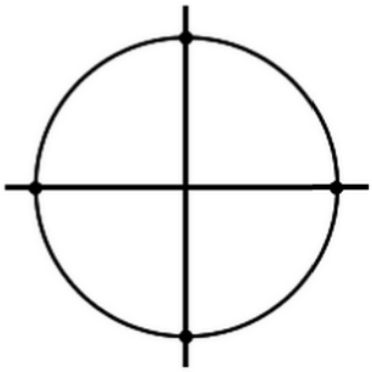
$\cos 0 = 1$ $\sec 0 = 1$

$\tan 0 = 0$ $\cot 0 =$ undefined



Evaluate the six trigonometric functions at the given value.

4.) $t = \pi$



Calculator Usage

round to thousandth

5.) $\csc\frac{\pi}{8}$

$\frac{1}{\sin\frac{\pi}{8}}$

$1 \div \sin(\pi/8)$

2.613

6.) $\sin\frac{2\pi}{3}$

$\sin(2\pi \div 3)$

0.866

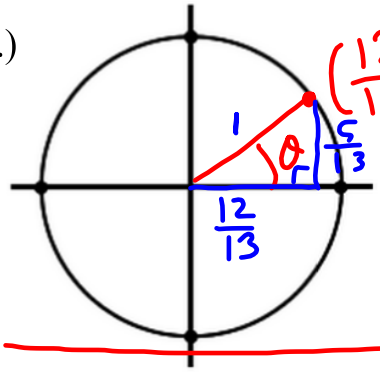
7.) $\cot 1.5$

$\frac{1}{\tan 1.5} = 0.071$

$\csc x \neq \sin^{-1} x$

Evaluate the six trigonometric functions at the given value.

8.)



$\left(\frac{12}{13}, \frac{5}{13}\right)$ ← is this on unit circle?

$\left(\frac{12}{13}\right)^2 + \left(\frac{5}{13}\right)^2 \stackrel{?}{=} 1$

$\frac{144}{169} + \frac{25}{169} = 1$

$\frac{169}{169} = 1$ ✓

$\sin\theta = \frac{5}{13}$

$\cos\theta = \frac{12}{13}$

$\tan\theta = \frac{5/\cancel{13}}{12/\cancel{13}}$

$\csc\theta = \frac{13}{5}$

$\sec\theta = \frac{13}{12}$

$\cot\theta = \frac{12}{5}$