Moore's law, a rule used in the computer industry, states that the number of transistors per integrated circuit (the processing power) doubles every year. Beginning in the early days of integrated circuits, the growth in capacity may be approximated by this table.

**Transistors per Integrated Chip**

| Year | 1965 | 1966 | 1967 | 1968 | 1969 | 1970 | 1971 |
|---|---|---|---|---|---|---|---|
| Transistors | 60 | 120 | 240 | 480 | 960 | 1920 | 3840 |

×2 ×2 ×2 ×2 ×2 ×2

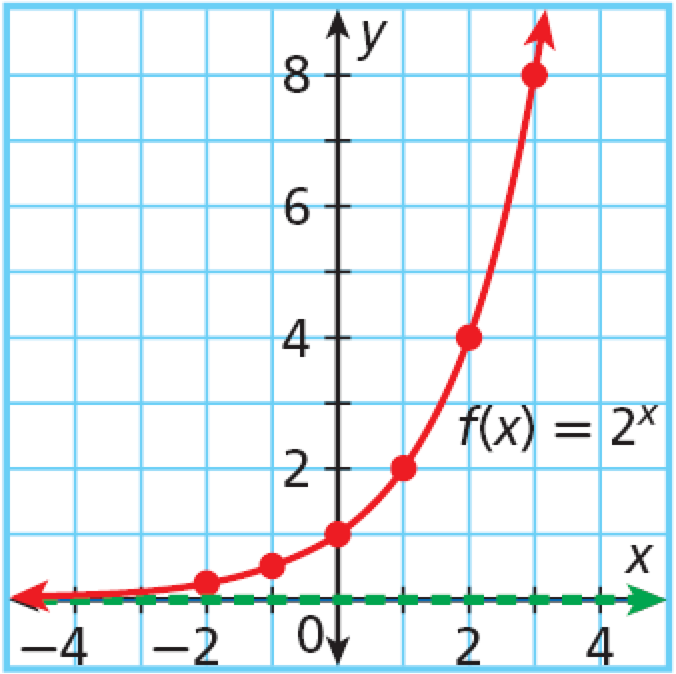
Growth that doubles every year can be modeled by using a function with a variable as an exponent. This function is known as an *exponential function*. The parent **exponential function** is $f(x) = b^x$, where the **base** $b$ is a constant and the exponent $x$ is the independent variable. **The domain is all real numbers and the range is {*y*|*y* > 0}.**

Range: $y > 0$

$(0, \infty)$

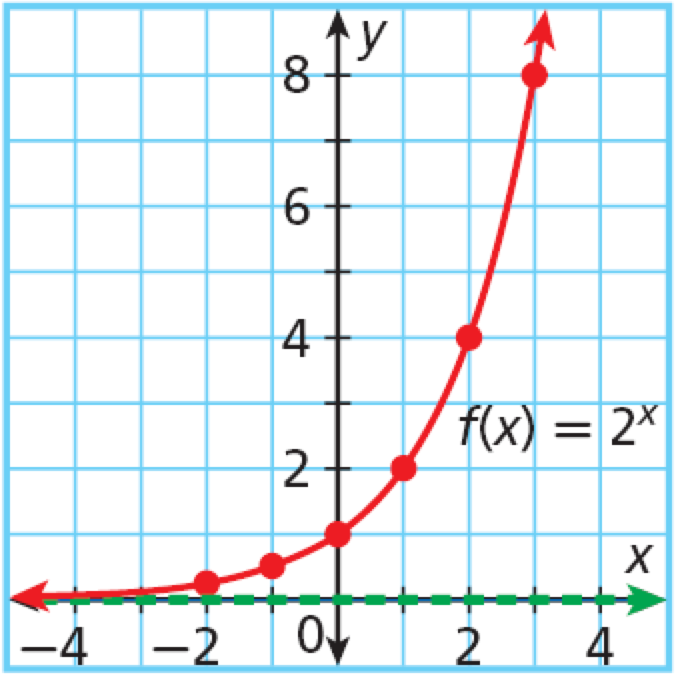
$$f(x) = \underset{\text{Base}}{b}^{\overset{\text{Exponent}}{x}}, \text{ where } b > 0,\ b \neq 1$$

The graph of the parent function $f(x) = 2^x$ is shown.

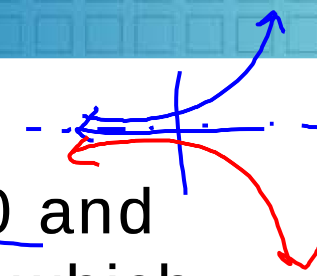



| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|---|
| $f(x) = 2^x$ | $\frac{1}{4}$ | $\frac{1}{2}$ | $1$ | $2$ | $4$ | $8$ |

Notice as the *x*-values decrease, the graph of the function gets closer and closer to the *x*-axis. The function never reaches the *x*-axis because the value of $2^x$ cannot be zero. In this case, the *x*-axis is an *asymptote*. An **asymptote** is a line that a graphed function approaches as the value of *x* gets very large or very small.

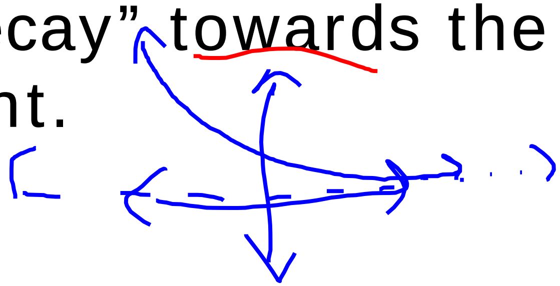


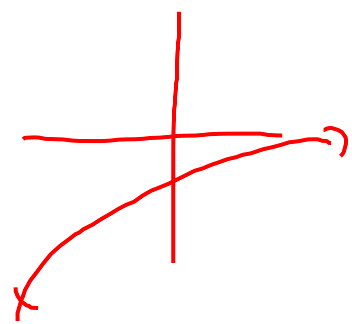
# 4-1 Exponential Functions, Growth, and Decay



A function of the form $\boxed{f(x) = ab^x,}$ with $a > 0$ and $b > 1$, is an **exponential growth** function, which increases as $x$ increases. The graph of an exponential growth function will "grow" ***away*** from the asymptote as you move left to right.

When $0 < b < 1$, the function is called an **exponential decay** function, which decreases as $x$ increases. Graphs of exponential decay functions "decay" towards the asymptote as you move left to right.





## Graphing Exponential Functions

To graph exponential functions, it's important to identify key points on the graph of $f(x) = ab^x$.

**Note: *a* is often referred to as the "initial value" of the function. Therefore, one key point on the graph of the function is (0, *a*).**

**Additional key points are: (1, *ab*), (2, $ab^2$) and**

$$\left(-1, \frac{a}{b}\right)$$

$\left(-1, \frac{a}{b}\right), (0, a), (1, ab), (2, ab^2)$

$\times b \quad \times b \quad \times b$

## Example 1A: Graphing Exponential Functions

**Tell whether the function shows growth or decay. Then graph.**

$$f(x) = 10\left(\frac{3}{4}\right)^x$$

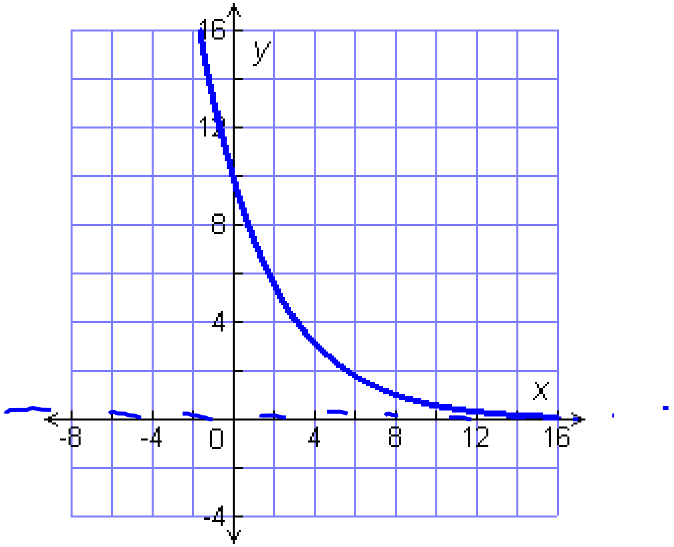
decay

$\frac{3}{4} < 1$

| $x$ | $f(x)$ |
|---|---|
| 0 | 10 |
| 1 | $\frac{15}{2}$ |
| 2 | $\frac{45}{8}$ |

## Example 1A Continued

**Step 2** Graph the function by using a table of values.



## Example 1B: Graphing Exponential Functions

**Tell whether the function shows growth or decay. Then graph.**

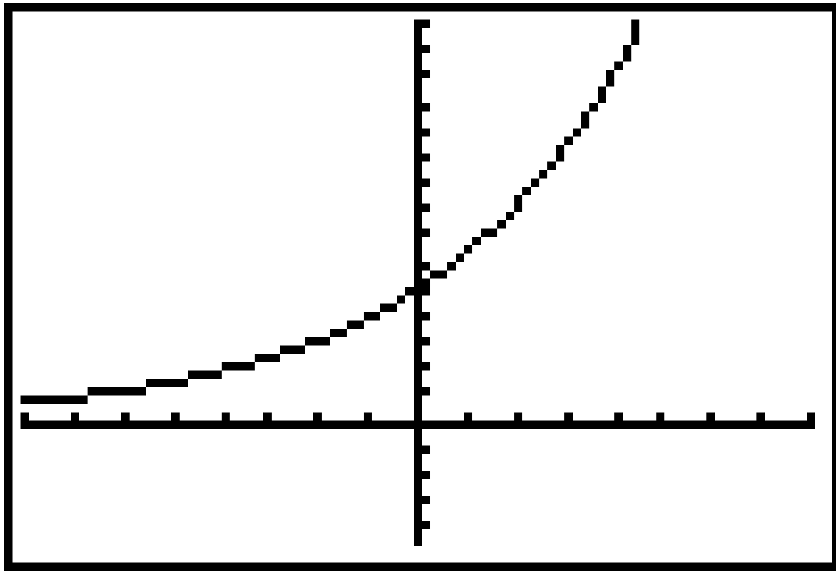
$g(x) = 100(1.05)^x$

growth

| x | g(x) |
|---|---|
| 0 | 100 |
| 1 | 105 |
| 2 | 110.25 |

## Example 1B Continued

**Step 2** Identify key points on the graph. Graph the function by using a graphing calculator.



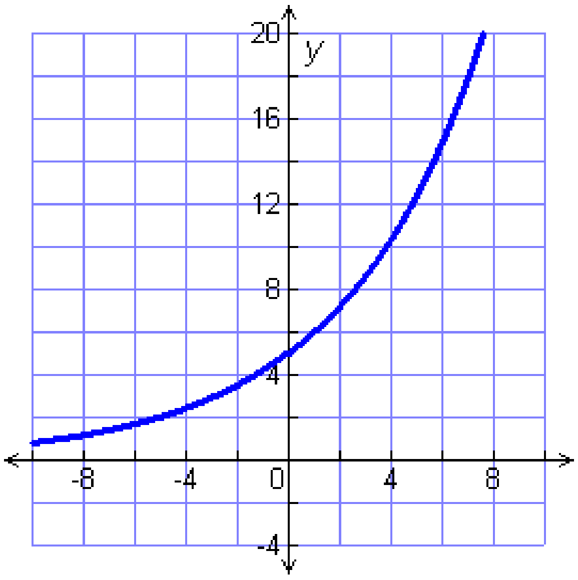
## Check It Out! Example 1

**Tell whether the function $p(x) = 5(1.2^x)$ shows growth or decay. Then graph.**

| $x$ | $p(x)$ |
|---|---|
| 0 | 5 |
| 1 | 6 |
| 2 | 7.2 |

## Check It Out! Example 1 Continued

**Step 2** Graph the function by using a table of values.



# 4-1 Exponential Functions, Growth, and Decay

You can model growth or decay by a constant percent increase or decrease with the following formula:

Initial amount — Number of time periods

$$A(t) = a(1 \pm r)^t$$

Final amount — Rate of increase /decrease

In the formula, the base of the exponential expression, $1 + r$, is called the *growth factor.* Similarly, $1 - r$ is the *decay factor*.

## Example 2: Economics Application

**Clara invests $5000 in an account that pays 6.25% interest per year. After how many years will her investment be worth $10,000?**

**Step 1** Write a function to model the growth in value of her investment.

$\overset{A}{\cancel{f}}(t) = a(1 + r)^t$ *Exponential growth function.*

$\overset{A}{\cancel{f}}(t) = 5000(1 + 0.0625)^t$ *Substitute 5000 for a and 0.0625 for r.*

$\overset{A}{\cancel{f}}(t) = 5000(1.0625)^t$ *Simplify.*

$$10000 = 5000(1.0625)^t$$

Use Graph to solve

$$Y_1 = 5000(1.0625)^x$$

$$Y_2 = 10000$$

$x = 11.43$

11.43 years

## Check It Out! Example 2

**In 1981, the Australian humpback whale population was 350 and increased at a rate of 14% each year since then. Write a function to model population growth. Use a graph to predict when the population will reach 20,000.**

$A(t) = 350(1.14)^t$

$t \approx 30.88$ years

## Example 3: Depreciation Application

**A city population, which was initially 15,500, has been dropping 3% a year. Write an exponential function and graph the function. Use the graph to predict when the population will drop below 8000.**

$A(t) = 15500(0.97)^t$

$t = 21.71$ years