3.7 INVESTIGATING GRAPHS OF POLYNOMIAL FUNCTIONS

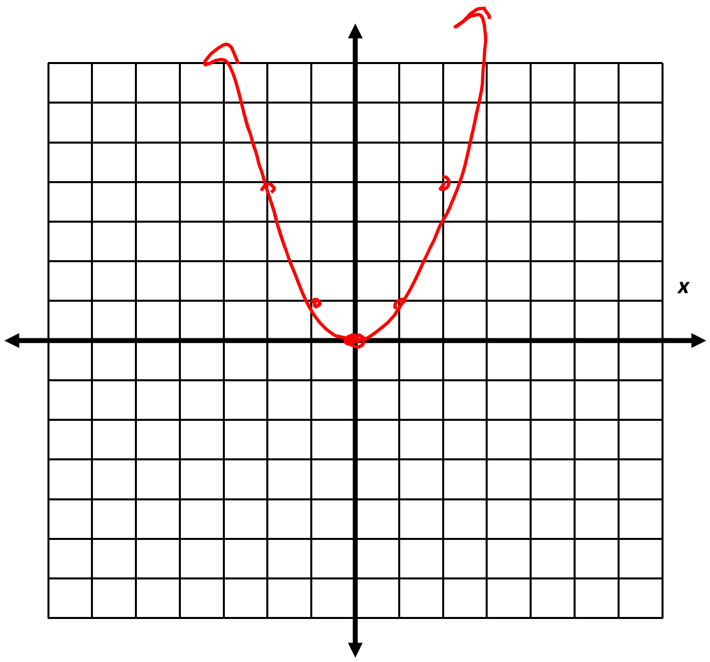
3.8 TRANSFORMING POLYNOMIAL FUNCTIONS

- Polynomial functions are classified by their degree.
- The graphs of polynomial functions are classified by the degree of the polynomial.
- Each graph, based on the degree, has a distinctive shape and characteristics.
- End behavior is a description of the values of the function as x approaches infinity ($x \longrightarrow +\infty$) or negative infinity ($x \longrightarrow -\infty$). It is helpful when you are graphing a polynomial function to know about the end behavior of the function.

y-values

- Let’s think about what we already know about some basic polynomial functions.

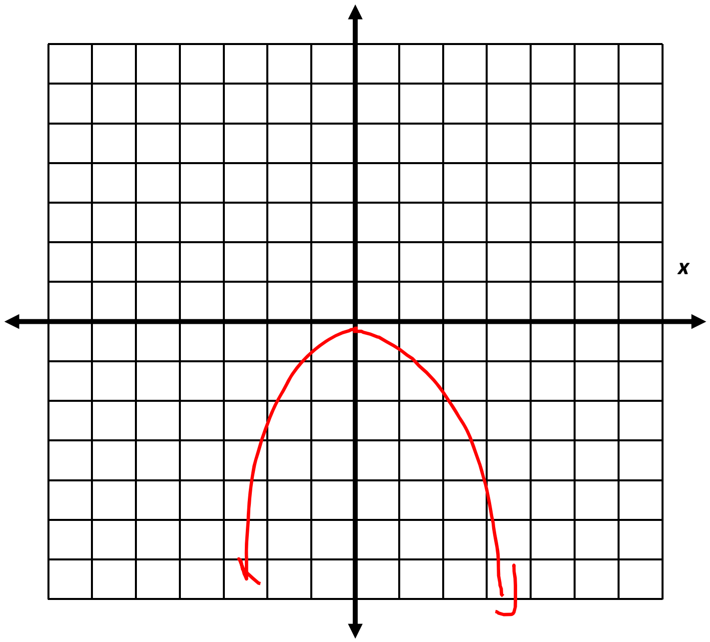
Sketch the graph of $f(x) = x^2$ and describe its end behavior.



as $x \to \infty$, $f(x) \to \infty$

as $x \to -\infty$, $f(x) \to \infty$

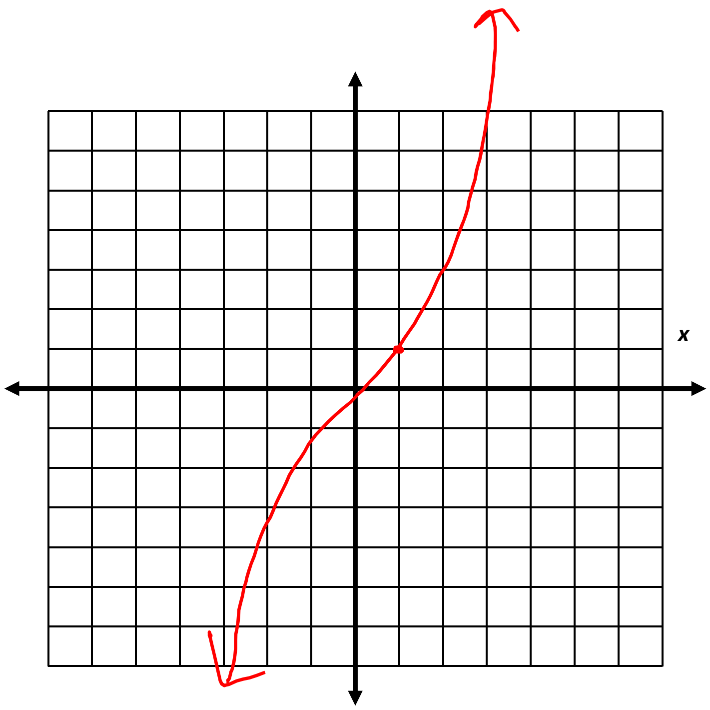
Sketch the graph of $f(x) = -x^2$ and describe its end behavior.



as $x \to \infty$, $f(x) \to -\infty$

as $x \to -\infty$, $f(x) \to -\infty$

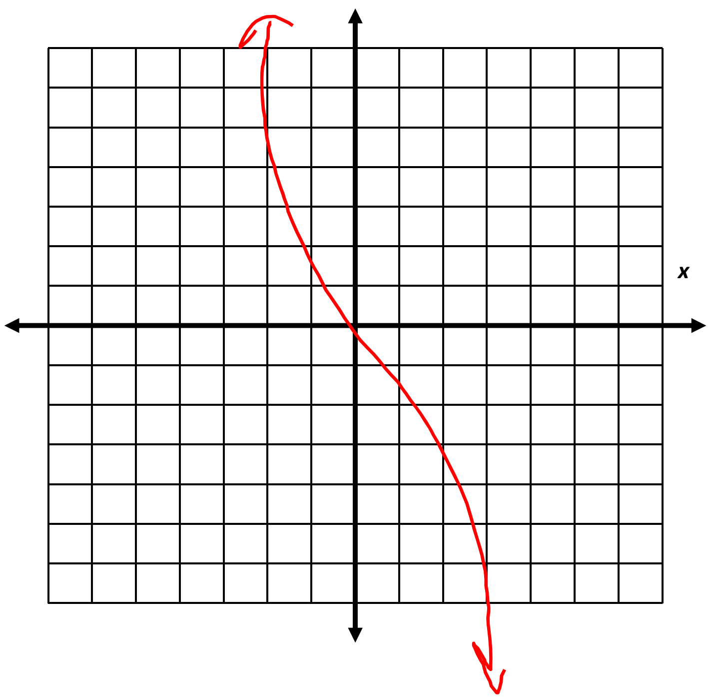
Sketch the graph of $f(x) = x^3$ and describe its end behavior.



as $x \to \infty$, $f(x) \to \infty$

as $x \to -\infty$, $f(x) \to -\infty$

Sketch the graph of $f(x) = -x^3$ and describe its end behavior.



as $x \to \infty$, $f(x) \to -\infty$

as $x \to -\infty$, $f(x) \to \infty$

Now explore the graphs of some other polynomial functions on your own, and make a conjecture about the characteristics of the function that seem to affect its end behavior. Write your thoughts/conjecture here, and then we'll summarize our findings together.

Function	End Behavior	
even degree positive leading coefficient	as $x \to \infty$, $f(x) \to \infty$	as $x \to -\infty$, $f(x) \to \infty$
even degree negative leading coefficient	as $x \to \infty$, $f(x) \to -\infty$	as $x \to -\infty$, $f(x) \to -\infty$
odd degree positive leading coef.	as $x \to \infty$, $f(x) \to \infty$	as $x \to -\infty$, $f(x) \to -\infty$
odd degree negative leading coef.	as $x \to \infty$, $f(x) \to -\infty$	as $x \to -\infty$, $f(x) \to \infty$

Example 1: Determining End Behavior of Polynomial Functions

Identify the leading coefficient, degree, and end behavior.

A. $Q(x) = -x^4 + 6x^3 - x + 9$

L.C. = -1

degree = 4

as $x \to \infty$, $f(x) \to -\infty$

as $x \to -\infty$, $f(x) \to -\infty$

B. $P(x) = 2x^5 + 6x^4 - x + 4$

L.C. = 2

degree : 5

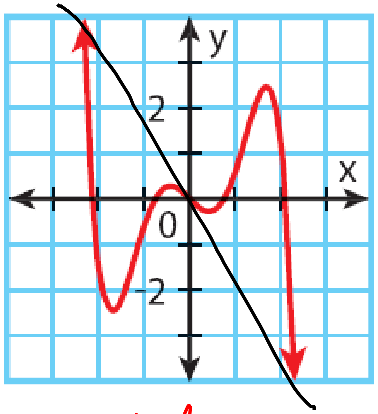
as $x \to \infty$, $f(x) \to \infty$

as $x \to -\infty$, $f(x) \to -\infty$

Example 2: Using Graphs to Analyze Polynomial Functions

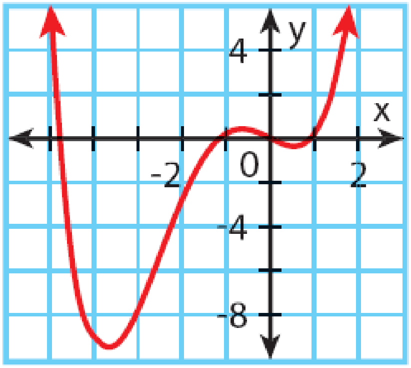
Identify whether the function graphed has an odd or even degree and a positive or negative leading coefficient.

A.



odd degree
negative lead. coef.

B.



even degree
positive leading coef.

Now that you have studied factoring, solving polynomial equations, and end behavior, you can graph a polynomial function.

Steps for Graphing a Polynomial Function
1. Find the real zeros and *y*-intercept of the function.
2. Plot the *x*- and *y*-intercepts.
3. Make a table for several *x*-values that lie between the real zeros.
4. Plot the points from your table.
5. Determine the end behavior of the graph.
6. Sketch the graph.

Example 3: Graphing Polynomial Functions

Graph the function. $f(x) = x^3 + 4x^2 + x - 6$.

possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 6$

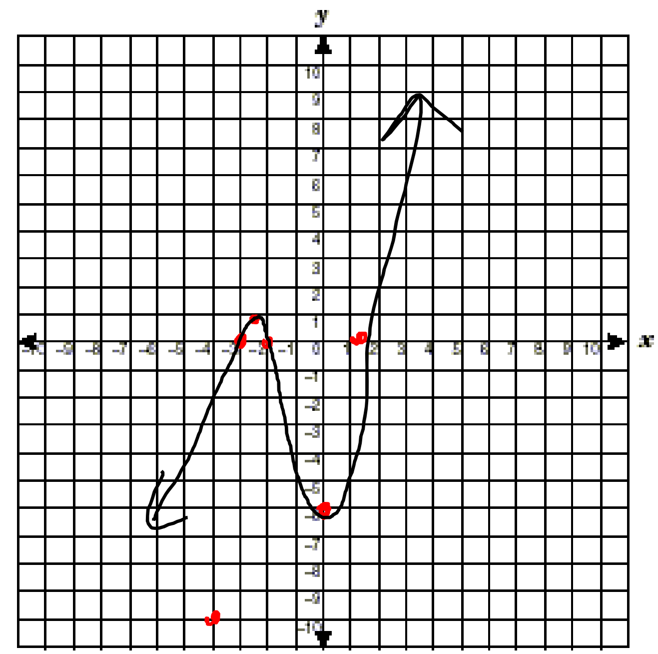
$$\begin{array}{c|cccc} 1 & 1 & 4 & 1 & -6 \\ & & 1 & 5 & 6 \\ \hline & 1 & 5 & 6 & 0 \end{array}$$

$x^2 + 5x + 6 = 0$

$(x+3)(x+2) = 0$

actual zeros: $-2, -3, 1$

y-int. $(0, -6)$



Check It Out! Example 3a

Graph the function. $f(x) = -x^3 + 2x^2 + 5x - 6$.