

A **turning point** is where a graph changes from increasing to decreasing or from decreasing to increasing. A turning point corresponds to a *local maximum* or *minimum*.

Local Maxima and Minima

For a function $f(x)$, $f(a)$ is a **local maximum** if there is an interval around a such that $f(x) < f(a)$ for every x -value in the interval except a .

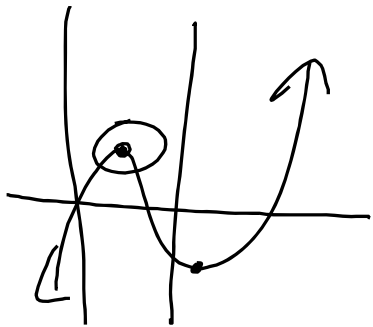
For a function $f(x)$, $f(a)$ is a **local minimum** if there is an interval around a such that $f(x) > f(a)$ for every x -value in the interval except a .

A polynomial function of degree n has ***at most*** $n - 1$ turning points and ***at most*** n x -intercepts.

You can use a graphing calculator to graph and estimate maximum and minimum values.

Example 4: Determine Maxima and Minima with a Calculator

A. Graph $f(x) = 2x^3 - 18x + 1$ on a calculator, and estimate the local maxima and minima.



local maximum 21.78 (occurs when $x = -1.73$)
local minimum -19.78 (occurs when $x = 1.73$)

B. Graph $g(x) = x^3 - 2x - 3$ on a calculator, and estimate the local maxima and minima.

You can perform the same transformations on polynomial functions that you performed on quadratic and linear functions.

Transformations of $f(x)$		
Transformation	$f(x)$ Notation	Examples
Vertical translation	$f(x) + k$	$g(x) = x^3 + 3$ 3 units up
		$g(x) = x^3 - 4$ 4 units down
Horizontal translation	$f(x - h)$	$g(x) = (x - 2)^3$ 2 units right
		$g(x) = (x + 1)^3$ 1 unit left
Vertical stretch/ compression	$af(x)$	$g(x) = 6x^3$ stretch by 6
		$g(x) = \frac{1}{2}x^3$ compression by $\frac{1}{2}$
Horizontal stretch/ compression	$f\left(\frac{1}{b}x\right)$	$g(x) = \left(\frac{1}{5}x\right)^3$ stretch by 5
		$g(x) = (3x)^3$ compression by $\frac{1}{3}$
Reflection	$-f(x)$	$g(x) = -x^3$ across x-axis
	$f(-x)$	$g(x) = (-x)^3$ across y-axis

Example 5: Translating a Polynomial Function

For $f(x) = x^3 - 6$, write the rule for each function and identify the transformation.

A. $g(x) = f(x) - 2$ $g(x) = x^3 - 8$
 $g(x) = (x^3 - 6) - 2$

vertical shift down 2

B. $h(x) = f(x + 3)$
 $h(x) = (x + 3)^3 - 6$

horizontal shift
left 3

Example 6: Reflecting a Polynomial Function

For $f(x) = x^3 + 5x^2 - 8x + 1$, write the rule for each function and identify the transformation.

A. $h(x) = \underline{-f(x)}$

$$h(x) = -(x^3 + 5x^2 - 8x + 1)$$

reflection in x-axis

$$h(x) = -x^3 - 5x^2 + 8x - 1$$

reflection in y-axis

B. $g(x) = \underline{f(-x)}$

$$g(x) = (-x)^3 + 5(-x)^2 - 8(-x) + 1$$

$$g(x) = -x^3 + 5x^2 + 8x + 1$$

$$g(x) = -x^3 + 5x^2 + 8x + 1$$