Examples: Solve each equation by finding all roots.

2. $x^4 + x^3 + 2x^2 + 4x - 8 = 0$ $x = 1,$

possible rational zeros: $\pm 1, \pm 2, \pm 4, \pm 8$

$$\begin{array}{c|ccccc} \underline{1} & 1 & 1 & 2 & 4 & -8 \\ & & 1 & 2 & 4 & 8 \\ \hline & 1 & 2 & 4 & 8 & 0 \end{array}$$

$(x^3 + 2x^2) + (4x + 8) = 0$

$x^2(x+2) + 4(x+2) = 0$

$$x^3 + 2x^2 + 4x + 8 = 0$$

$$(x+2)(x^2+4) = 0$$

$$x = -2 \qquad x^2 + 4 = 0$$

$$x^2 = -4$$

$$x = \pm 2i$$

Completely
Factored Form

$$(x-1)(x+2)(x+2i)(x-2i) = 0$$

$$x = 1, -2, \pm 2i$$

Examples: Solve each equation by finding all roots.

Solutions: $\frac{1}{2}$, $2 \pm \sqrt{6}$

3. $2x^3 - 9x^2 + 2 = 0$

Factored Form: $2(x - \frac{1}{2})(x - (2 + \sqrt{6}))(x - (2 - \sqrt{6})) = 0$

Possible rational zeros:
$$\frac{\text{factors of constant}}{\text{factors of lead. coef}} = \frac{\pm 1, \pm 2}{\pm 1, \pm 2} = \pm 1, \frac{\pm \frac{1}{2} \pm 2}{1}$$

$\frac{1}{2}$ ⌋ 2   -9   0   2
              1   -4   -2
         _____
         2 -8   -4 | 0

$2x^2 - 8x - 4 = 0$
$2(x^2 - 4x - 2) = 0$

$x = 2 \pm \sqrt{6}$

Write the simplest polynomial function with the given zeros.

$$0, -4, \sqrt{3}, -\sqrt{3}$$

$$f(x) = x(x+4)(x-\sqrt{3})(x+\sqrt{3})$$

$$= x(x+4)(x^2-3)$$

$$= x(x^3+4x^2-3x-12)$$

$$f(x) = x^4+4x^3-3x^2-12x$$

Write the simplest function with zeros $2i, 1+\sqrt{2}$, and 3.

$-2i, 1-\sqrt{2}$

$f(x) = (x-3)(x-2i)(x+2i)(x-(1+\sqrt{2}))(x-(1-\sqrt{2}))$

$f(x) = (x-3)(x^2+4)((x-1)^2 - 2)$

$(x^2 - 2x + 1 - 2)$

$f(x) = (x^3 - 3x^2 + 4x - 12)(x^2 - 2x - 1)$

$f(x) = x^5 - 5x^4 + 9x^3 - 17x^2 + 20x + 12$

Lesson Wrap Up

Write the simplest polynomial function with the given zeros.

8. 2, −1, 1

9. 0, −2, $\sqrt{3}$

⑨ $f(x) = x^4 + 2x^3 - 3x^2 - 6x$

⑩ $f(x) = x^4 + x^3 + 2x^2 + 4x - 8$

10. 2i, 1, −2

11. Solve by finding all roots.

$x^4 - 5x^3 + 7x^2 - 5x + 6 = 0$