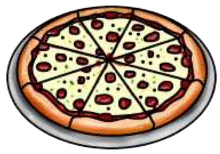

3-3 Dividing Polynomials

I. Dividing a Polynomial by a Monomial

This process is similar to the Distributive Property, only you're dividing instead of multiplying. In other words, divide each term of the polynomial by the monomial.

Examples

(1) $\dfrac{12x^5 - 36x^3 + 4x^2}{2x} = \dfrac{12x^5}{2x} - \dfrac{36x^3}{2x} + \dfrac{4x^2}{2x} = 6x^4 - 18x^2 + 2x$

(2) $\dfrac{15a^2b^3 - 25ab^2 + 10a^3b}{5ab} = 3ab^2 - 5b + 2a^2$

II. Dividing a Polynomial by a Polynomial

We will learn two methods for dividing polynomials by polynomials. The first is long division. Before we start, Let's review long division for real numbers. For example, let's do 171 ÷ 3 using long division.

```
   57
3 |171
  -15↓
   21
  -21
```

Now, your turn. Do 2302 ÷ 12 using long division.

```
    191 10/12      191 5/6
12 |2302
   -12
    110
   -108
      22
     -12
      10
```

Now, let's see how it works for polynomials.

A. Long Division

Divide the following using long division.

(1) $(x^2 - 7x + 10) \div (x - 2)$

$x - 5$

$x - 2 \,\overline{)\, x^2 - 7x + 10}$

$-x^2 + 2x$

$-5x + 10$

$+5x + {}^{-}10$

0

insert placeholder $0x^2$

(2) $(2x^4 + 3x^3 + 5x - 1) \div (x^2 - 2x + 2)$

$2x^2 + 7x + 10 + \frac{11x - 21}{x^2 - 2x + 2}$

$x^2 - 2x + 2 \,\overline{)\, 2x^4 + 3x^3 + 0x^2 + 5x - 1}$

$-\cancel{2x^4} + 4x^3 + {}^{-}4x^2$

$7x^3 - 4x^2 + 5x$

$-7x^3 + 14x^2 + {}^{-}14x$

$10x^2 - 9x - 1$

$-10x^2 + 20x + {}^{-}20$

$11x - 21$

Try these on your own.

Divide the following using long division.

(3) $(x^2 + 9x + 14) \div (x + 7)$

(4) $(x^2 + 7x - 5) \div (x - 2)$

$x + 9 + 13/x - 2$

B. Synthetic Division

This method can only be used when dividing by a linear binomial whose leading coefficient is 1. In other words, this method can only be used when dividing by a polynomial of the form (x - k).

Let's see how it works.

Divide the following using synthetic division.

k

(1) $\left(x^3+2x^2-6x-9\right)\div\left(x+3\right)$

$$\begin{array}{r|rrrr} -3 & 1 & 2 & -6 & -9 \\ & & -3 & 3 & 9 \\ \hline & 1 & -1 & -3 & 0 \end{array}$$

multiply

add

coefficients of dividend

$x^2 - x - 3$

Divide the following using synthetic division.

(2) $\left(x^3 \; 0x^2 -7x-6\right)\div\left(x-2\right)$

$x^2+2x-3-{}^{12}/_{x-2}$

$$\begin{array}{r|rrrr} 2 & 1 & 0 & -7 & -6 \\ & & 2 & 4 & -6 \\ \hline & 1 & 2 & -3 & -12 \end{array}$$

(3) $\left(4x^2+5x-4\right)\div\left(x+1\right)$

$$\begin{array}{r|rrr} -1 & 4 & 5 & -4 \\ & & -4 & -1 \\ \hline & 4 & 1 & -5 \end{array}$$

$4x+1-{}^{5}/_{x+1}$

Remainder Theorem

If the polynomial function $P(x)$ is divided by $x - a$, then the remainder r is $P(a)$.

Example:

If your last name begins with A - L, find the value of $P(3)$ if

$P(x) = x^3 - 4x^2 + 5x + 1$.

$P(3) = 3^3 - 4(3)^2 + 5(3) + 1 = 7$

If your last name begins with M - Z, use synthetic division to divide $x^3 - 4x^2 + 5x + 1$ by $x - 3$.

$$
\begin{array}{r|rrrr}
3 & 1 & -4 & 5 & 1 \\
  &   & 3 & -3 & 6 \\
\hline
  & 1 & -1 & 2 & 7
\end{array}
$$

$x^2 - x + 2 + \frac{7}{x-3}$

Use synthetic substitution to evaluate the polynomial for the given value.

(1) $P(x) = x^3 - 4x^2 + 3x - 5$ for $x = 4$

Find $P(4)$

$$
\begin{array}{r|rrrr}
4 & 1 & -4 & 3 & -5 \\
  &   & 4 & 0 & 12 \\
\hline
  & 1 & 0 & 3 & 7
\end{array}
$$

$\boxed{P(4) = 7}$

(2) $P(x) = 4x^4 + 2x^3 + 3x + 5$ for $x = -½$

$$
\begin{array}{r|rrrrr}
-1/2 & 4 & 2 & 0 & 3 & 5 \\
  &   & -2 & 0 & 0 & -3/2 \\
\hline
  & 4 & 0 & 0 & 3 & 7/2
\end{array}
$$

$P(-1/2) = 7/2$