



### 3-3 Dividing Polynomials

#### I. Dividing a Polynomial by a Monomial

This process is similar to the Distributive Property, only you're dividing instead of multiplying. In other words, divide each term of the polynomial by the monomial.

#### Examples

$$(1) \frac{12x^5 - 36x^3 + 4x^2}{2x} = \frac{12x^5}{2x} - \frac{36x^3}{2x} + \frac{4x^2}{2x} = 6x^4 - 18x^2 + 2x$$

$$(2) \frac{15a^2b^3 - 25ab^2 + 10a^3b}{5ab} = 3ab^2 - 5b + 2a^2$$

#### II. Dividing a Polynomial by a Polynomial

We will learn two methods for dividing polynomials by polynomials. The first is long division. Before we start, Let's review long division for real numbers. For example, let's do  $171 \div 3$  using long division.

$$\begin{array}{r} 57 \\ 3 \overline{) 171} \\ \underline{-15} \phantom{0} \\ 21 \\ \underline{-21} \\ 0 \end{array}$$

Now, your turn. Do  $2302 \div 12$  using long division.

$$\begin{array}{r} 191^{10} / 12 \quad 191^5 / 6 \\ 12 \overline{) 2302} \\ \underline{-12} \phantom{00} \\ 110 \phantom{0} \\ \underline{-108} \phantom{0} \\ 22 \\ \underline{-24} \\ 10 \end{array}$$

Now, let's see how it works for polynomials.

### A. Long Division

Divide the following using long division.

(1)  $(x^2 - 7x + 10) \div (x - 2)$

$$\begin{array}{r} x-5 \\ x-2 \overline{) x^2 - 7x + 10} \\ \underline{-x^2 + 2x} \phantom{0} \\ -5x + 10 \\ \underline{+5x - 10} \\ 0 \end{array}$$

(2)  $(2x^4 + 3x^3 + 5x - 1) \div (x^2 - 2x + 2)$

insert placeholder  $0x^2$

$$\begin{array}{r} 2x^2 + 7x + 10 + \frac{11x-21}{x^2-2x+2} \\ x^2-2x+2 \overline{) 2x^4 + 3x^3 + 0x^2 + 5x - 1} \\ \underline{-2x^4 + 4x^3 + 4x^2} \phantom{0} \\ 7x^3 - 4x^2 + 5x \\ \underline{-7x^3 + 14x^2 + 14x} \phantom{0} \\ 10x^2 - 9x - 1 \\ \underline{-10x^2 + 20x + 20} \\ 11x - 21 \end{array}$$

Try these on your own.

Divide the following using long division.

(3)  $(x^2 + 9x + 14) \div (x + 7)$

(4)  $(x^2 + 7x - 5) \div (x - 2)$

$$x + 9 + \frac{13}{x-2}$$

**B. Synthetic Division**

*This method can only be used when dividing by a linear binomial whose leading coefficient is 1. In other words, this method can only be used when dividing by a polynomial of the form  $(x - k)$ .*

Let's see how it works.

Divide the following using synthetic division.

(1)  $(x^3 + 2x^2 - 6x - 9) \div (x + 3)$

Handwritten synthetic division for  $(x^3 + 2x^2 - 6x - 9) \div (x + 3)$ . The divisor is  $-3$ . The dividend coefficients are  $1, 2, -6, -9$ . The process shows multiplying  $-3$  by  $1$  to get  $-3$ , adding to  $2$  to get  $-1$ , multiplying  $-3$  by  $-1$  to get  $3$ , adding to  $-6$  to get  $-3$ , multiplying  $-3$  by  $-3$  to get  $9$ , and adding to  $-9$  to get  $0$ . The quotient is  $x^2 - x - 3$ . Annotations include "coefficients of dividend" pointing to the top row and "multiply" pointing to the first step.

Divide the following using synthetic division.

(2)  $(x^3 - 7x - 6) \div (x - 2)$

$x^2 + 2x - 3 - 12/x - 2$

Handwritten synthetic division for  $(x^3 - 7x - 6) \div (x - 2)$ . The divisor is  $2$ . The dividend coefficients are  $1, 0, -7, -6$ . The process shows multiplying  $2$  by  $1$  to get  $2$ , adding to  $0$  to get  $2$ , multiplying  $2$  by  $2$  to get  $4$ , adding to  $-7$  to get  $-3$ , multiplying  $2$  by  $-3$  to get  $-6$ , and adding to  $-6$  to get  $-12$ .

(3)  $(4x^2 + 5x - 4) \div (x + 1)$

$4x + 1 - 5/x + 1$

Handwritten synthetic division for  $(4x^2 + 5x - 4) \div (x + 1)$ . The divisor is  $-1$ . The dividend coefficients are  $4, 5, -4$ . The process shows multiplying  $-1$  by  $4$  to get  $-4$ , adding to  $5$  to get  $1$ , multiplying  $-1$  by  $1$  to get  $-1$ , and adding to  $-4$  to get  $-5$ .

**Remainder Theorem**

If the polynomial function  $P(x)$  is divided by  $x - a$ , then the remainder  $r$  is  $P(a)$ .

Example:

If your last name begins with A - L, find the value of  $P(3)$  if

$$P(x) = x^3 - 4x^2 + 5x + 1.$$

$$P(3) = 3^3 - 4(3)^2 + 5(3) + 1 = 7$$

If your last name begins with M - Z, use synthetic division to divide  $x^3 - 4x^2 + 5x + 1$  by  $x - 3$ .

$$\begin{array}{r|rrrr} 3 & 1 & -4 & 5 & 1 \\ & & 3 & -3 & 6 \\ \hline & 1 & -1 & 2 & 7 \end{array}$$

$x^2 - x + 2 + \frac{7}{x-3}$

**Use synthetic substitution to evaluate the polynomial for the given value.**

(1)  $P(x) = x^3 - 4x^2 + 3x - 5$  for  $x = 4$

$$\begin{array}{r|rrrr} 4 & 1 & -4 & 3 & -5 \\ & & 4 & 0 & 12 \\ \hline & 1 & 0 & 3 & 7 \end{array}$$

Find  $P(4)$

$$P(4) = 7$$

(2)  $P(x) = 4x^4 + 2x^3 + 3x + 5$  for  $x = -\frac{1}{2}$

$$\begin{array}{r|rrrrr} -\frac{1}{2} & 4 & 2 & 0 & 3 & 5 \\ & & -2 & 0 & 0 & -\frac{3}{2} \\ \hline & 4 & 0 & 0 & 3 & \frac{7}{2} \end{array}$$

$$P(-\frac{1}{2}) = \frac{7}{2}$$