Warm Up

Enter the following keystrokes on a calculator.

$\sqrt{\ }$ (−) 1 ENTER

Describe what the calculator gives you as a solution.
Repeat the keystrokes with a different negative number.
Why does the calculator give this answer?

3.2 Complex Numbers

i: imaginary unit

$i = \sqrt{-1}$

Core Concept

The Square Root of a Negative Number

Property	Example
1. If r is a positive real number, then $\sqrt{-r} = i\sqrt{r}$.	$\sqrt{-3} = i\sqrt{3}$
2. By the first property, it follows that $(i\sqrt{r})^2 = -r$.	$(i\sqrt{3})^2 = i^2 \cdot 3 = -3$

* since $i = \sqrt{-1}$, $i^2 = (\sqrt{-1})^2 = -1$

Find the square root of each number. Simplify!

a. $\sqrt{-25}$ $= i\sqrt{25}$ $5i$

b. $\sqrt{-72}$ $= i\sqrt{72}$ $i\sqrt{36}\sqrt{2}$ $6i\sqrt{2}$

c. $-5\sqrt{-9}$ $= -5i\sqrt{9}$ $-5i(3)$ $-15i$

Find the square root of the number.

d. $\sqrt{-4}$ $2i$

e. $\sqrt{-12}$ $2i\sqrt{3}$

f. $-\sqrt{-36}$ $= -i\sqrt{36}$ $-6i$ $6(-i)$

g. $2\sqrt{-54}$ $= 2i\sqrt{54}$ $\sqrt{9}\sqrt{6}$ $6i\sqrt{6}$ $2(3i\sqrt{6})$

(2) Find the values of x and y that satisfy the equation.

a. $2x - 7i = 10 + yi$.

* real parts must be =
@ imaginary parts be =

* $2x = 10$

$x = 5$

@ $-7i = yi$

$-7 = y$

Find the values of x and y that satisfy the equation.

b. $x + 3i = 9 - yi$

$x = 9$

$3 = -y$

$y = -3$

c. $9 + 4yi = -2x + 3i$

$\frac{9}{-2} = \frac{-2x}{-2}$

$x = -9/2$

$4y = 3$

$y = 3/4$

Core Concept

Sums and Differences of Complex Numbers

To add (or subtract) two complex numbers, add (or subtract) their real parts and their imaginary parts separately.

Sum of complex numbers: $(a + bi) + (c + di) = (a + c) + (b + d)i$

Difference of complex numbers: $(a + bi) - (c + di) = (a - c) + (b - d)i$

(3) Add or subtract. Write the answer in standard form.

a. $(8 - i) + (5 + 4i) = 13 + 3i$

b. $(7 - 6i) - (3 - 6i) = 4$

$-6i + (+6i) = 0$

$7 - 6i - 3 + 6i$

c. $13 - (2 + 7i) + 5i$

$13 - 2 - 7i + 5i$

$11 + 12i$

$11 + 2i$

$11 - 2i$

(4) Multiply. Write the answer in standard form.

a. $4i(-6 + i)$

$-24i + 4i^2$

$-24i + 4(-1)$

never leave i^2 in final answer

$-4 - 24i$

b. $(9 - 2i)(-4 + 7i)$ $a + bi$

$-36 + 63i + 8i - 14i^2$

$-36 + 63i + 8i + 14$

$-22 + 71i$

(5) Perform the operation. Write the answer in standard form.

a. $(9 - i) + (-6 + 7i)$

$3 + 6i$

b. $(3 + 7i) - (8 - 2i)$

$-5 + 9i$

c. $-4 - (1 + i) - (5 + 9i)$

$-4 - 1 - i - 5 - 9i$

$-10 - 10i$

d. $(-3i)(10i) = -30i^2$

30 $-30(-1)$

e. $i(8 - i)$ $8i - i^2$

$8i + 1$

$1 + 8i$

f. $(3 + i)(5 - i)$ $+1$

$15 - 3i + 5i - i^2$

$16 + 2i$

(6) Solve each equation.

(a) $x^2 + 4 = 0$ → $\sqrt{x^2} = \pm\sqrt{-4}$ → $x = \pm 2i$

(b) $2x^2 - 11 = -47$

$+11 \quad +11$

$\frac{2x^2}{2} = \frac{-36}{2}$

$\sqrt{x^2} = \sqrt{-18}$

$x = \pm\sqrt{-18}$

$x = \pm i\sqrt{18}$

$\sqrt{9}\sqrt{2}$

$x = \pm 3i\sqrt{2}$

(7) Find the zeros of $f(x) = 4x^2 + 20$.

$4x^2 + 20 = 0$

$4x^2 = -20$

$\sqrt{x^2} = \sqrt{-5}$

$x = \pm i\sqrt{5}$

Mixed Practice

Solve the equation.

1. $x^2 = -13$ 2. $x^2 + 11 = 3$ 3. $3x^2 - 7 = -31$

Find the zeros of the function.

4. $f(x) = x^2 + 7$ 5. $f(x) = -x^2 - 4$ 6. $f(x) = 9x^2 + 1$