3.1 Solving Quadratic Equations by Graphing, Using Square Roots, and Factoring

The Basics:

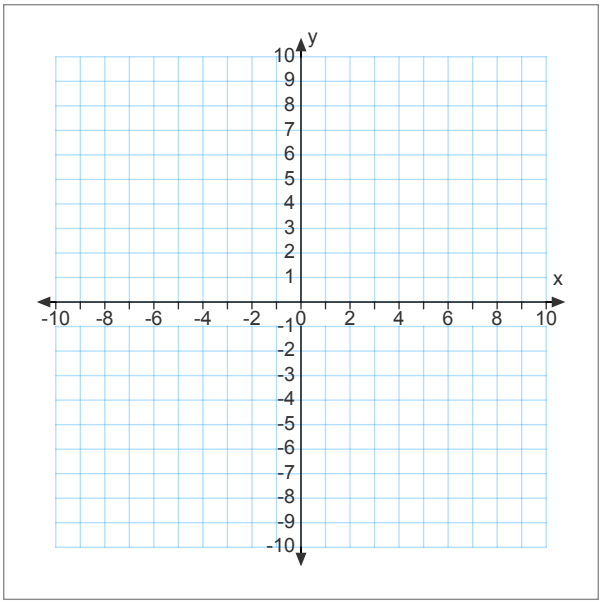
A quadratic equation in one variable is an equation that can be written in the standard form $ax^2 + bx + c = 0$, where a, b, and c are real numbers and $a \neq 0$. A root of an equation is a solution of the equation.

I. Solving Quadratic Equations by Graphing

Find the x-intercepts of the related function $y = ax^2 + bx + c$

Example #1

Find the zeros of $f(x) = -x^2 + 6x - 8$ by using a graph and a table.



II. Solving Quadratic Equations by Finding Square Roots

Write the equation in the form $u^2 = d$, where u is an algebraic expression, and solve by taking the square root of each side. Remember to account for the positive and negative square root.

Example #2

Solve each equation using square roots.

a) $3x^2 - 4 = 71$

$\frac{3x^2}{3} = \frac{75}{3}$

$\sqrt{x^2} = \pm\sqrt{25}$

$x = \pm 5$

$x = 5, x = -5$

$\{-5, 5\}$

b) $\sqrt{(x-3)^2} = \pm\sqrt{16}$

$x - 3 = \pm 4$

$+3 \quad +3$

$x = 3 \pm 4$

$x = 7, -1$

c) $4x^2 + 3 = 11$

$\frac{4x^2}{4} = \frac{8}{4}$

$\sqrt{x^2} = \pm\sqrt{2}$

$x = \pm\sqrt{2}$

d) $\frac{2(x-5)^2}{2} = \frac{54}{2}$

$\sqrt{(x-5)^2} = \pm\sqrt{27}$

$x - 5 = \pm\sqrt{27}$

$\sqrt{9}\sqrt{3}$

$x - 5 = \pm 3\sqrt{3}$

$x = 5 \pm 3\sqrt{3}$

Additional Examples

Solve each equation using square roots.

3. $\sqrt{(x+6)^2} = \pm\sqrt{28}$ $x+6 = \pm 2\sqrt{7}$ $x = -6 \pm 2\sqrt{7}$

4. $x^2 - 49 = 0$ $\sqrt{x^2} = \pm\sqrt{49}$ $x = \pm 7$

5. $-2x^2 = -72$ $\frac{-2}{-2}$ $\sqrt{x^2} = \pm\sqrt{36}$ $x = \pm 6$

III. Solving Quadratic Equations by Factoring

Zero Product Property: $p \cdot q = 0$ $p = 0$ or $q = 0$

For all real numbers *a* and *b*, if *ab* = 0, then *a* = 0 or *b* = 0.

***You can use the Zero Product Property to solve some quadratic equations by factoring.**

Example #6

Find the zeros of each function by factoring.

a) $f(x) = x^2 - 8x + 12$

$x^2 - 8x + 12 = 0$

$(x - 6)(x - 2) = 0$

$x - 6 = 0$ $\quad x = 6$

$x - 2 = 0$ $\quad x = 2$

b) $g(x) = 3x^2 + 12x$

$3x^2 + 12x = 0$

$3x(x + 4) = 0$

$3x = 0$ $\quad x = 0$

$x + 4 = 0$ $\quad x = -4$

c) $h(x) = x^2 - 5x + 6$

Example #7

Find the roots of each equation by factoring.

a) $28x = 4x^2 - 72$

b) $9x^2 = 1$

Additional Examples - Each problem should be solved by factoring.

8. Find the roots of the following equation: $5x^2 + 20 = 20x$

9. Find the zeros of the following function: $g(x) = 9x^2 - x$

10. Find the zeros of the following function: $h(x) = x^2 - 13x + 14$

11. Find the dimensions of a rectangle with an area of 210 cm^2 and whose length is one more than its width.