Factoring Polynomials

Recall that if a number is divided by any of its factors, the remainder is 0. Likewise, if a polynomial is divided by any of its factors, the remainder is 0.

The Remainder Theorem states that if a polynomial is divided by $(x - a)$, the remainder is the value of the function at a. So, if $(x - a)$ is a factor of $P(x)$, then $P(a) = 0$.

Factor Theorem

THEOREM	EXAMPLE
For any polynomial $P(x)$, $(x - a)$ is a factor of $P(x)$ if and only if $P(a) = 0$.	Because $P(1) = 1^2 - 1 = 0$, $(x - 1)$ is a factor of $P(x) = x^2 - 1$.

Factoring Polynomials

Example 1: Determining Whether a Linear Binomial is a Factor

Determine whether the given binomial is a factor of the polynomial $P(x)$.

A. $(x + 2)$; $(4x^2 - 2x + 5)$

NO b/c $P(-2) \neq 0$

$-2 \mid 4 \quad -2 \quad 5$

$\quad\quad -8 \quad 20$

$4 \quad -10 \quad 25$ ← will this = 0?

B. $(3x - 6)$; $(3x^4 - 6x^3 + 6x^2 + 3x - 30)$

*divide everything by 3 $(x-2)$; $(x^4 - 2x^3 + 2x^2 + x - 10)$

$2 \mid 1 \quad -2 \quad 2 \quad 1 \quad -10$

$\quad\quad 2 \quad 0 \quad 4 \quad 10$

$1 \quad 0 \quad 2 \quad 5 \mid 0$

yes b/c $P(2) = 0$

Check It Out! Example 1

Determine whether the given binomial is a factor of the polynomial $P(x)$.

A. $(x + 1)$; $(x^2 - 3x + 1)$

$-1 \mid 1 \quad -3 \quad 1$
$\qquad\quad -1 \quad 4$
$1 \quad -4 \quad 5$

No b/c $P(-1) \neq 0$

b/c remainder $\neq 0$

B. $(x + 2)$; $(3x^4 + 6x^3 - 5x - 10)$

Yes b/c $P(-2) = 0$

Write $P(x)$ in factored form:

$P(x) = (3x^3 - 5)(x + 2)$

$-2 \mid 3 \quad 0 \quad -5 \quad -10$

Factoring Polynomials

Example 2: Factoring by Grouping

Factor: $x^3 - x^2 - 25x + 25$.

$x^3 - x^2 - 25x + 25$

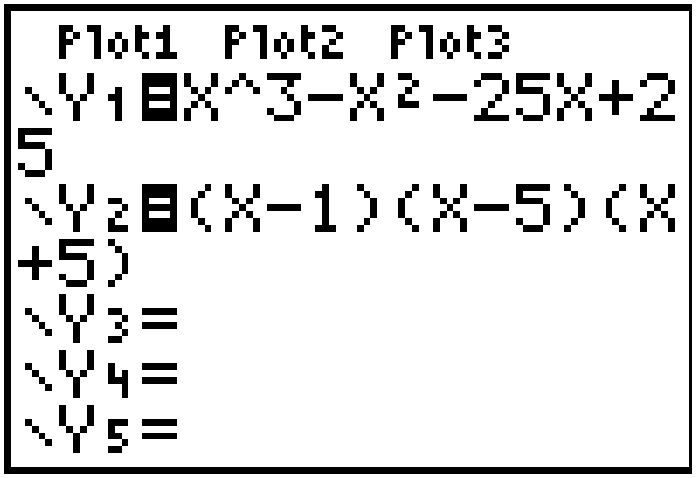
$x^2(x-1) - 25(x-1)$

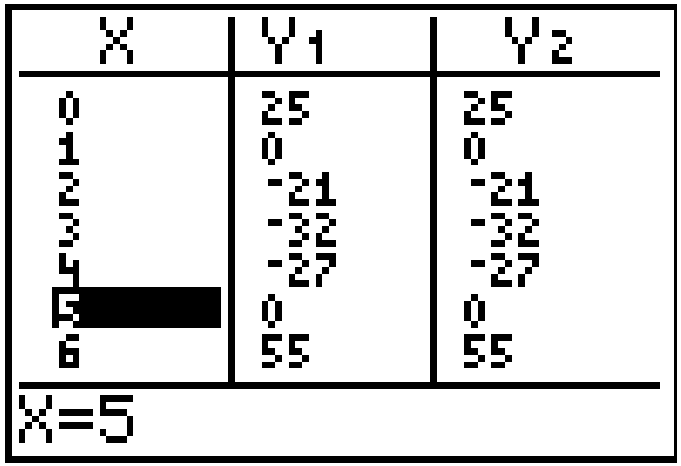
$(x-1)(x^2-25)$

$(x-1)(x+5)(x-5)$

Example 2 Continued

Check Use the table feature of your calculator to compare the original expression and the factored form.





The table shows that the original function and the factored form have the same function values. ✓

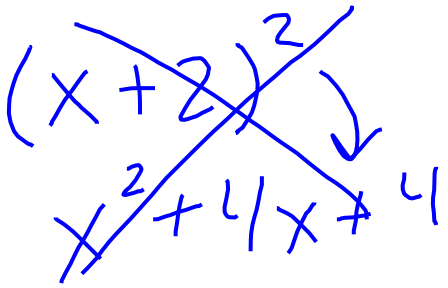
Check It Out! Example 2a

Factor: $x^3 - 2x^2 - 9x + 18$.

$x^2(x-2) - 9(x-2)$

$(x^2-9)(x-2)$

$(x+3)(x-3)(x-2)$



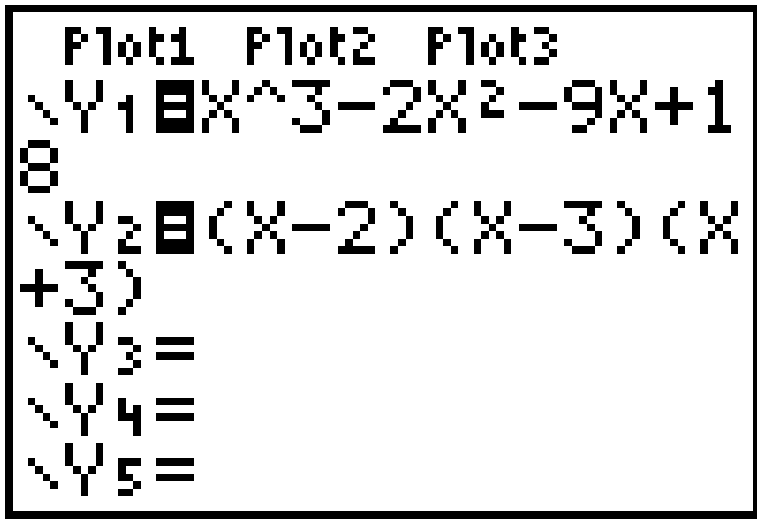
Factor $2x^3 + x^2 + 8x + 4$

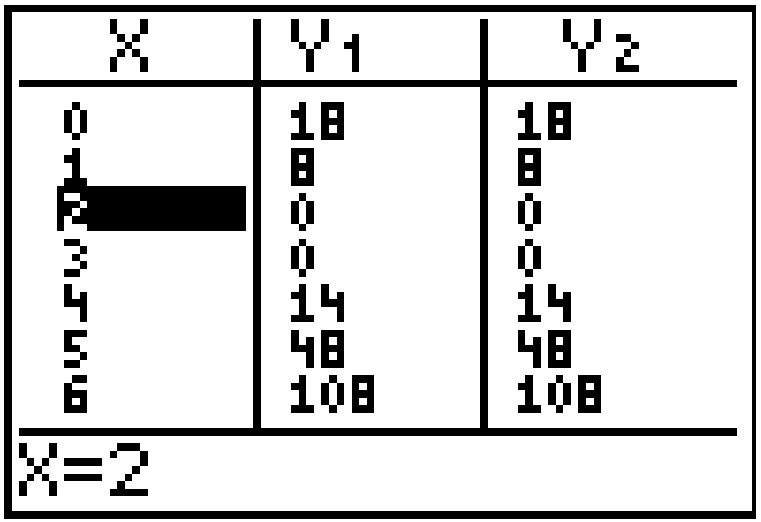
$x^2(2x+1) + 4(2x+1)$

$(x^2+4)(2x+1)$

Check It Out! Example 2a Continued

Check Use the table feature of your calculator to compare the original expression and the factored form.





The table shows that the original function and the factored form have the same function values. ✓

Check It Out! Example 2b

Factor: $2x^3 + x^2 + 8x + 4$.

Factoring Polynomials

Just as there is a special rule for factoring the difference of two squares, there are special rules for factoring the sum or difference of two cubes.

Factoring the Sum and the Difference of Two Cubes

METHOD	ALGEBRA
Sum of two cubes	$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
Difference of two cubes	$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Example 3A: Factoring the Sum or Difference of Two Cubes

Factor the expression. $a = x$ $b = 3$

$4x^4 + 108x$ $(x)^3 + (3)^3$

$4x(x^3 + 27)$

$4x(x + 3)(x^2 - 3x + 9)$

Example 3B: Factoring the Sum or Difference of Two Cubes

Factor the expression.

$125d^3 - 8$

$a = 5d \quad b = 2$

$(5d - 2)(25d^2 + 10d + 4)$

Check It Out! Example 3a

Factor the expression.

$8 + z^6 = (2 + z^2)(4 - 2z^2 + z^4)$

Check It Out! Example 3b

Factor the expression.

$2x^5 - 16x^2$

$2x^2(x-2)(x^2+2x+4)$