

2.4 Modeling with Quadratic Functions

Core Concept

Writing Quadratic Equations

Given a point and the vertex (h, k)

Use vertex form:

$$y = a(x - h)^2 + k$$

Given a point and x -intercepts p and q

Use intercept form:

$$y = a(x - p)(x - q)$$

Given three points

Write and solve a system of three equations in three variables.

$$y = ax^2 + bx + c$$

1. The graph shows the parabolic path of a performer who is shot out of a cannon, where y is the height (in feet) and x is the horizontal distance traveled (in feet).

a. Write an equation of the parabola.

$$y = a(x - h)^2 + k$$

$$15 = a(0 - 50)^2 + 35$$

$$15 = 2500a + 35$$

$$-35$$

$$-35$$

$$-20 = 2500a$$

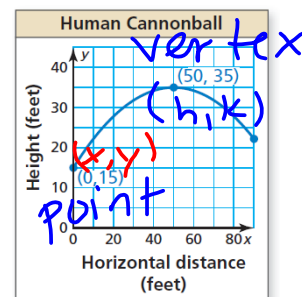
$$a = \frac{-20}{2500} = -\frac{1}{125}$$

$$y = -\frac{1}{125}(x - 50)^2 + 35$$

b. The performer lands in a net 90 feet from the cannon. What is the height of the net?

$$y = -\frac{1}{125}(90 - 50)^2 + 35$$

22.2 feet



2. Write an equation of the parabola that passes through the point $(-1, 2)$ and has vertex $(4, -9)$. (h, k)

(x, y)

$$2 = a(-1-4)^2 - 9$$

$$2 = a(-5)^2 - 9$$

$$2 = 25a - 9$$

$$11 = 25a$$

$$a = \frac{11}{25}$$

$$y = \frac{11}{25}(x-4)^2 - 9$$

3. A meteorologist creates a parabola to predict the temperature tomorrow, where x is the number of hours after midnight and y is the temperature (in degrees Celsius).

a. Write a function f that models the temperature over time.

$$y = \frac{1}{10}(x-4)(x-24)$$

$$y = a(x-p)(x-q)$$

$$9.6 = a(0-4)(0-24)$$

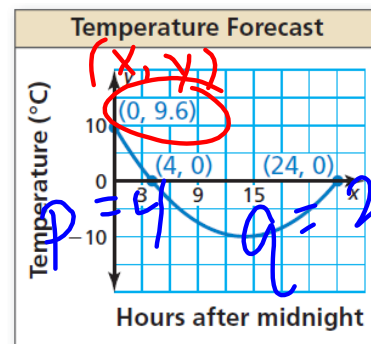
$$9.6 = a(-4)(-24)$$

b. What is the coldest temperature?

Find the vertex

$$\frac{p+q}{2} = 14$$

$$(14, -10)$$



$$9.6 = a(-4)(-24)$$

$$a = \frac{1}{10}$$

4. Write an equation of the parabola that passes through the point (2, 5) and has x-intercepts -2 and 4 .

$$(x, y) \quad 5 = a(2+2)(2-4)$$

$$y = a(x-p)(x-q)$$

$$y = a(x+2)(x-4)$$

$$5 = a(4)(-2)$$

$$5 = -8a$$

$$a = -5/8$$

$$y = -\frac{5}{8}(x+2)(x-4)$$

5. NASA can create a weightless environment by flying a plane in parabolic paths. The table shows heights h (in feet) of a plane t seconds after starting the flight path. After about 20.8 seconds, passengers begin to experience a weightless environment. Write and evaluate a function to approximate the height at which this occurs.

Time, t	Height, h
10	26,900
15	29,025
20	30,600
25	31,625
30	32,100
35	32,025
40	31,400

① Writing the function

$$y = ax^2 + bx + c$$

$$(10, 26900)$$

$$26900 = a(10^2) + b(10) + c$$

$$(15, 29025)$$

$$29025 = a(15^2) + b(15) + c$$

$$(20, 30600)$$

$$30600 = a(20^2) + b(20) + c$$

$$\begin{aligned} \longrightarrow & (26900 = 100a + 10b + c) \\ \longrightarrow & 29025 = 225a + 15b + c \\ \longrightarrow & 30600 = 400a + 20b + c \end{aligned}$$

$$26900 = 100(-11) + 10(700) + c$$

$$26900 = -1100 + 7000 + c \quad \leftarrow$$

$$26900 = 5900 + c$$

$$\boxed{c = 21000}$$

$$\begin{array}{r|l} \begin{array}{l} -26900 = -100a - 10b - c \\ + 29025 = 225a + 15b + c \\ \hline 2125 = 125a + 5b \end{array} & \begin{array}{l} -26900 = -100a - 10b - c \\ + 30600 = 400a + 20b + c \\ \hline 3700 = 300a + 10b \end{array} \end{array}$$

$$\begin{array}{r}
 -2(2125 = 125a + 5b) \\
 3700 = 300a + 10b \\
 -4250 = -250a - 10b \\
 \hline
 -550 = 50a \\
 a = -11 \\
 2125 = 125(-11) + 5b \\
 2125 = -1375 + 5b \\
 3500 = 5b \\
 b = 700 \\
 \boxed{y = -11x^2 + 700x + 21000}
 \end{array}$$

6. Write an equation of the parabola that passes through the points $(-1, 4)$, $(0, 1)$, and $(2, 7)$.

$$\begin{array}{l}
 a(-1)^2 + b(-1) + c = 4 \rightarrow a - b + c = 4 \\
 a(0)^2 + b(0) + c = 1 \\
 a(2)^2 + b(2) + c = 7
 \end{array}$$

$c = 1$
 $4a + 2b + c = 7$

$$\begin{array}{l}
 a - b + 1 = 4 \rightarrow a - b = 3 \\
 4a + 2b + 1 = 7 \rightarrow 4a + 2b = 6 \\
 + 2a - 2b = 6 \\
 \hline
 6a = 12 \quad a = 2 \\
 2 - b = 3 \\
 -b = 1 \quad b = -1
 \end{array}$$

$$\boxed{y = 2x^2 - x + 1}$$