2.2

Characteristics of Quadratic Functions For use with Exploration 2.2

Essential Question What type of symmetry does the graph of

 $f(x) = a(x - h)^2 + k$ have and how can you describe this symmetry?

EXPLORATION: Parabolas and Symmetry

Work with a partner.

a. Complete the table. Then use the values in the table to sketch the graph of the function

$$f(x) = \frac{1}{2}x^2 - 2x - 2$$
 on graph paper.

| | - |
|-----------------------|---|
| <i>f</i> (<i>x</i>) | |

| x | 3 | 4 | 5 | 6 |
|-----------------------|---|---|---|---|
| <i>f</i> (<i>x</i>) | | | | |



- **b.** Use the results in part (a) to identify the vertex of the parabola.
- **c.** Find a vertical line on your graph paper so that when you fold the paper, the left portion of the graph coincides with the right portion of the graph. What is the equation of this line? How does it relate to the vertex?



d. Show that the vertex form $f(x) = \frac{1}{2}(x-2)^2 - 4$ is equivalent to the function given in part (a).

2.2 Characteristics of Quadratic Functions (continued)



EXPLORATION: Parabolas and Symmetry

Work with a partner. Repeat Exploration 1 for the function given by $f(x) = -\frac{1}{3}x^2 + 2x + 3 = -\frac{1}{3}(x - 3)^2 + 6$.

| x | -2 | -1 | 0 | 1 | 2 |
|-----------------------|----|----|---|---|---|
| <i>f</i> (<i>x</i>) | | | | | |

| x | 3 | 4 | 5 | 6 |
|-----------------------|---|---|---|---|
| <i>f</i> (<i>x</i>) | | | | |



Communicate Your Answer

3. What type of symmetry does the graph of the parabola $f(x) = a(x - h)^2 + k$ have and how can you describe this symmetry?

4. Describe the symmetry of each graph. Then use a graphing calculator to verify your answer.

a.
$$f(x) = -(x-1)^2 + 4$$
 b. $f(x) = (x+1)^2 - 2$ **c.** $f(x) = 2(x-3)^2 + 1$

d.
$$f(x) = \frac{1}{2}(x+2)^2$$
 e. $f(x) = -2x^2 + 3$ **f.** $f(x) = 3(x-5)^2 + 2$

2.2 Notetaking with Vocabulary For use after Lesson 2.2

In your own words, write the meaning of each vocabulary term.

axis of symmetry

standard form

minimum value

maximum value

intercept form

Core Concepts

Properties of the graph of $f(x) = ax^2 + bx + c$



- The parabola opens up when a > 0 and open down when a < 0.
- The graph is narrower than the graph of $f(x) = x^2$ when |a| > 1 and wider when |a| < 1.

• The axis of symmetry is
$$x = -\frac{b}{2a}$$
 and the vertex is $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.

• The y-intercept is c. So, the point (0, c) is on the parabola.

Notes:

2.2 Notetaking with Vocabulary (continued)

Minimum and Maximum Values

For the quadratic function $f(x) = ax^2 + bx + c$, the y-coordinate of the vertex is the **minimum value** of the function when a > 0 and the **maximum value** when a < 0.



- Minimum value: $f\left(-\frac{b}{2a}\right)$
- Domain: All real numbers
- Range: $y \ge f\left(-\frac{b}{2a}\right)$
- Decreasing to the left of $x = -\frac{b}{2a}$
- Increasing to the right of $x = -\frac{b}{2a}$

Notes:



- Maximum value: $f\left(-\frac{b}{2a}\right)$
- Domain: All real numbers

• Range:
$$y \le f\left(-\frac{b}{2a}\right)$$

• Increasing to the left of
$$x = -\frac{b}{2a}$$

• Decreasing to the right of
$$x = -\frac{b}{2a}$$

Properties of the graph of f(x) = a(x - p)(x - q)

- Because f(p) = 0 and f(q) = 0, p and q are the x-intercepts of the graph of the function.
- The axis of symmetry is halfway between (p, 0) and (q, 0). So, the axis of symmetry is $x = \frac{p+q}{2}$.
- The parabola opens up when a > 0 and opens down when a < 0.



Notes:

2.2 Notetaking with Vocabulary (continued)

Extra Practice

In Exercises 1–3, graph the function. Label the vertex and axis of symmetry. Find the minimum or maximum value of the function. Describe the domain and range of the function, and where the function is increasing and decreasing.



In Exercises 4 and 5, graph the function. Label the *x*-intercept(s), vertex, and axis of symmetry.

- 4. f(x) = 4(x + 4)(x 3)5. f(x) = -7x(x - 6)5. f(x) = -7x(x - 6)
- 6. A softball player hits a ball whose path is modeled by $f(x) = -0.0005x^2 + 0.2127x + 3$, where x is the distance from home plate (in feet) and y is the height of the ball above the ground (in feet). What is the highest point this ball will reach? If the ball was hit to center field which has an 8 foot fence located 410 feet from home plate, was this hit a home run? Explain.