Essential Question

What type of symmetry does the graph of $f(x) = a(x - h)^2 + k$ have and how can you describe this symmetry?

line symmetry

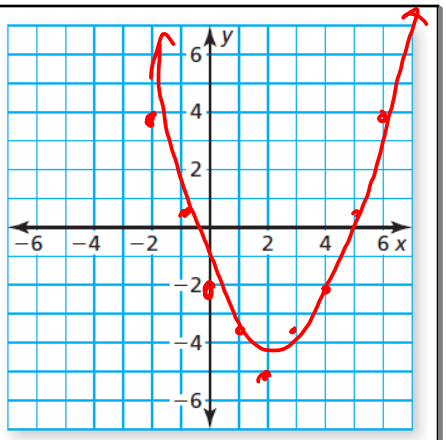
Axis of symmetry: $x = h$

Work with a partner.

a. Complete the table. Then use the values in the table to sketch the graph of the function

$$f(x) = \frac{1}{2}x^2 - 2x - 2$$

on graph paper.



x	−2	−1	0	1	2
f(x)	4	½	-2	-3½	4

x	3	4	5	6
f(x)	-3½	-2	½	4

$\frac{1}{2}(-2)^2 - 2(-2) - 2$

b. Use the results in part (a) to identify the vertex of the parabola.

$(2, -4)$

c. Find a vertical line on your graph paper so that when you fold the paper, the left portion of the graph coincides with the right portion of the graph. What is the equation of this line? How does it relate to the vertex?

$x = 2$

d. Show that the vertex form

$$f(x) = \frac{1}{2}(x-2)^2 - 4$$

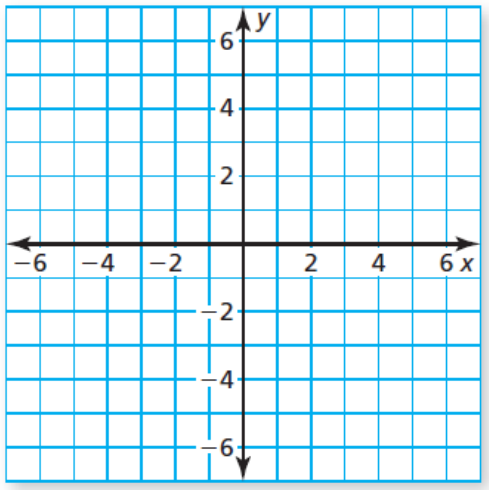
is equivalent to the function given in part (a).

x	$\frac{1}{2}(x-2)^2 - 4$
-2	4
-1	$\frac{1}{2}$
0	-2
1	

$\frac{1}{2}(-2-2)^2 - 4$

Work with a partner. Repeat Exploration 1 for the function given by

$$f(x) = -\frac{1}{3}x^2 + 2x + 3 = -\frac{1}{3}(x-3)^2 + 6$$



Communicate Your Answer

3. What type of symmetry does the graph of the parabola $f(x) = a(x - h)^2 + k$ have and how can you describe this symmetry?

line symmetry

Write equation for axis of symmetry: $x = h$

4. Describe the symmetry of each graph. Then use a graphing calculator to verify your answer.

a. $f(x) = -(x - 1)^2 + 4$ $x = 1$

b. $f(x) = (x + 1)^2 - 2$ $x = -1$

c. $f(x) = 2(x - 3)^2 + 1$ $x = 3$

d. $f(x) = \frac{1}{2}(x + 2)^2$ $x = -2$

e. $f(x) = -2x^2 + 3$ $x = 0$

f. $f(x) = 3(x - 5)^2 + 2$ $x = 5$

In your own words, write the meaning of each vocabulary term.

axis of symmetry

standard form

minimum value

maximum value

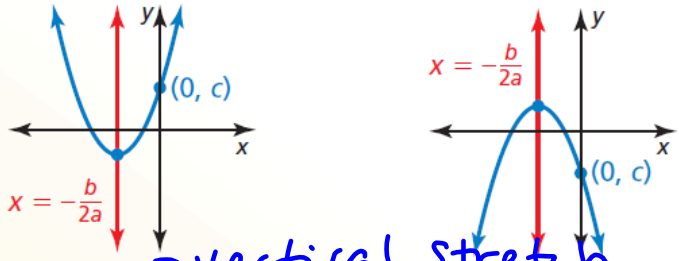
intercept form

Core Concept

Standard Form

Properties of the Graph of $f(x) = ax^2 + bx + c$

$y = ax^2 + bx + c, a > 0$ $y = ax^2 + bx + c, a < 0$

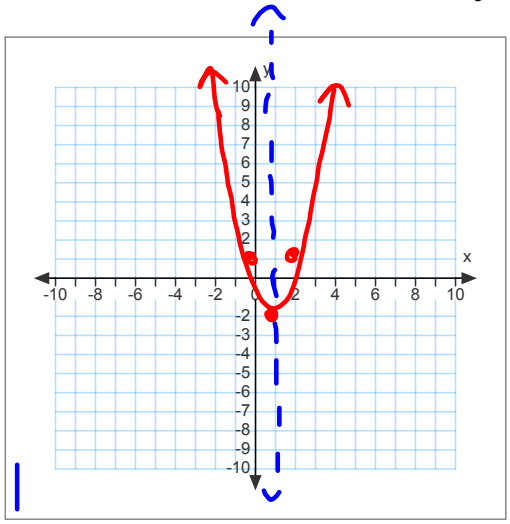


- The parabola opens up when $a > 0$ and opens down when $a < 0$.
- The graph is narrower than the graph of $f(x) = x^2$ when $|a| > 1$ and wider when $|a| < 1$.
- The axis of symmetry is $x = -\frac{b}{2a}$ and the vertex is $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.
- The y-intercept is c. So, the point $(0, c)$ is on the parabola.

vertical stretch

vertical compression

Graph $f(x) = 3x^2 - 6x + 1$. Label the vertex and axis of symmetry



* graphs of quadratic functions need at least 3 pts: AOS

$a = 3$ $x = \frac{-b}{2a} = \frac{6}{2(3)} = 1$

$b = -6$ V: (1, -2)

$c = 1$ y-intercept (0, 1)

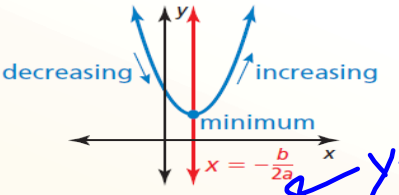
$\frac{-b}{2a}$

Core Concept

Minimum and Maximum Values

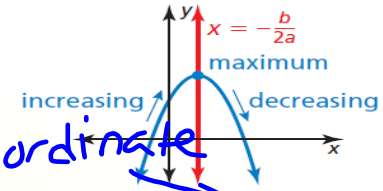
For the quadratic function $f(x) = ax^2 + bx + c$, the y-coordinate of the vertex is the **minimum value** of the function when $a > 0$ and the **maximum value** when $a < 0$.

$a > 0$



- Minimum value: $f\left(-\frac{b}{2a}\right)$
- Domain: All real numbers
- Range: $y \geq f\left(-\frac{b}{2a}\right)$
- Decreasing to the left of $x = -\frac{b}{2a}$
- Increasing to the right of $x = -\frac{b}{2a}$

$a < 0$



- Maximum value: $f\left(-\frac{b}{2a}\right)$
- Domain: All real numbers
- Range: $y \leq f\left(-\frac{b}{2a}\right)$
- Increasing to the left of $x = -\frac{b}{2a}$
- Decreasing to the right of $x = -\frac{b}{2a}$

y-coordinate of vertex

$(-\infty, \infty)$

$\left[f\left(-\frac{b}{2a}\right), \infty\right)$

$\left(-\infty, f\left(-\frac{b}{2a}\right)\right]$

Find the minimum value or maximum value of $f(x) = \frac{1}{2}x^2 - 2x - 1$.

Describe the domain and range of the function, and where the function is increasing and decreasing.

$a = \frac{1}{2}$

$b = -2$

$c = -1$

Vertex $(2, -3)$

$-\frac{b}{2a} = \frac{2}{2(\frac{1}{2})} = \frac{2}{1} = 2$

minimum value: -3 (y-value)

D: $(-\infty, \infty)$

R: $[-3, \infty)$

increasing: right of $x = 2$

decreasing: left of $x = 2$