

14.2 Arithmetic and Geometric Sequences

Two common sequences that appear frequently in mathematics are the arithmetic and geometric sequences.

Arithmetic Sequence

An **arithmetic sequence** is one in which the same number is **added** or **subtracted** from each term to get the next term in the **sequence**. The number you add or subtract is called the **common difference**.

Handy rules involving an arithmetic sequence:

$$\begin{aligned} a_1 &= a_1 \\ a_2 &= a_1 + d \\ a_3 &= a_1 + 2d \\ a_4 &= a_1 + 3d \\ a_5 &= a_1 + 4d \\ &\vdots \end{aligned}$$

Nth term of an Arithmetic Sequence

The n th term of an arithmetic sequence with first term a_1 and common difference d is given by:

$$a_n = a_1 + (n - 1)d$$

Example 1. Are the following arithmetic sequences?

3, 8, 13, 18, 23, 28...

-2, -12, -22, -32, ...

2, 4, 8, 16, 32, ...

14, 14.5, 15, 15.5, 16...

Example 2. Given two terms in an arithmetic sequence, find the common difference, the 52nd term, and the explicit formula.

$$a_{20} = 70$$

$$a_{33} = 96$$

Write your questions and thoughts here!

Time-out of for some math history....



Sum of a Finite Arithmetic Sequence

The sum of the first n terms of an arithmetic sequence is given by:

$$S_n = \frac{n(a_1 + a_n)}{2}$$

Example 3: Find the sum of the first 20 terms of the arithmetic series:

- a. $2 + 6 + 10 + 14 + 18 + \dots$
- b. Suppose the sum of the series has a sum of 2178. Find n such that $S_n = 2178$.

Geometric Sequence

A **geometric sequence** is one in which the same number is **multiplied** or **divided** by each term to get the next term in the **sequence**. The number you multiply or divide by is called the **common ratio**, usually denoted by r .

Determine if the following sequences are arithmetic, geometric or neither.

4. 1, 2, 6, 24, 120, ...
5. 81, 27, 9, 3, 1, ...
6. 5, 10, 15, 20, 25, ...

Nth term of a Geometric Sequence

The n th term of a geometric sequence with first term a_1 and common ratio r is given by:

$$a_n = a_1 r^{n-1}$$

Example 7: The third term of a geometric series equals 64 while the common ratio is 2.

- a. Write a rule for the n th term.
- b. Find the 9th term

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Write your questions and thoughts here!



Sum of a Finite Geometric Series

The n th term of a geometric sequence with first term a_1 and common ratio r is given by:

$$S_n = a_1 \left(\frac{1 - r^n}{1 - r} \right)$$

Example 8: Find the sum of the first 10 terms of the series $1 + 5 + 25 + 125 + \dots$

Example 9: For which term would $S_n = 3906$?

Now summarize what you learned!


