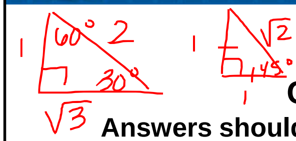
10-3 The Unit Circle



Warm Up

Complete the table below.

Answers should be exact and given in simplest radical form.

θ (in degrees)	30°	45°	60°
θ (in radians)	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$\sin\theta$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
$\tan\theta$	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

10-3 The Unit Circle

WS

Angles + Angle Measure

#9

$$-154°47'42'' = -154.795°$$

$$47' \cdot \frac{1°}{60'} = 0.7833°$$

$$42'' \cdot \frac{1°}{3600''} = 0.01167$$

Holt McDougal Algebra 2
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10-3 The Unit Circle

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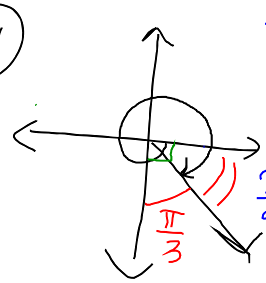
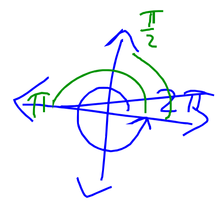
$-\frac{13\pi}{6}$





$\frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$

$2\frac{1}{6}\pi$

$\frac{3\pi}{6} - \frac{2\pi}{6} = \frac{\pi}{6}$

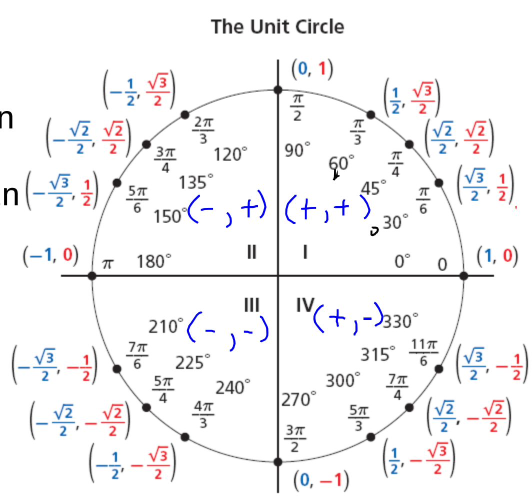
10-3 The Unit Circle

A **unit circle** is a circle with a radius of 1 unit. For every point $P(x, y)$ on the unit circle, the value of r is 1. Therefore, for an angle θ in the standard position:

$$\sin\theta = \frac{y}{r} = \frac{y}{1} = y$$

$$\cos\theta = \frac{x}{r} = \frac{x}{1} = x$$

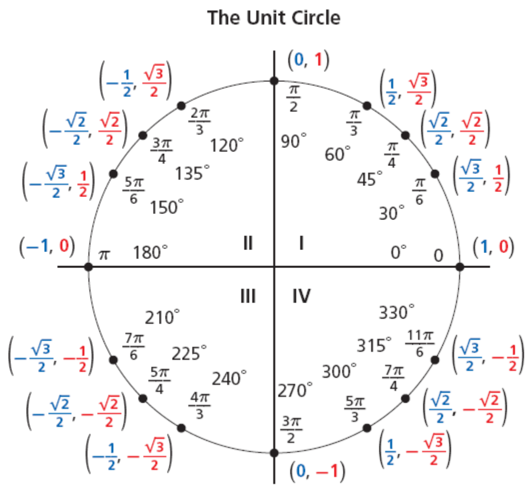
$$\tan\theta = \frac{y}{x}$$



10-3 The Unit Circle

So the coordinates of *P* can be written as ($\cos\theta$, $\sin\theta$).

The diagram shows the equivalent degree and radian measure of special angles, as well as the corresponding *x*- and *y*-coordinates of points on the unit circle.



10-3 The Unit Circle

Example 1: Using the Unit Circle to Evaluate Trigonometric Functions

Use the unit circle to find the exact value of each trigonometric function.

$\cos 225° = -\frac{\sqrt{2}}{2}$

$\tan \frac{5\pi}{6} = \frac{y}{x} = \frac{\sin\theta}{\cos\theta} = \frac{1/\cancel{2}}{-\sqrt{3}/\cancel{2}} = \frac{1}{-\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

$\frac{1}{\cancel{2}} \cdot -\frac{\cancel{2}}{\sqrt{3}}$

10-3 The Unit Circle

Check It Out! Example 1a

Use the unit circle to find the exact value of each trigonometric function.

sin 315° $= -\frac{\sqrt{2}}{2}$

tan 180° $= 0$

$\cos\frac{4\pi}{3} = -\frac{1}{2}$

10-3 The Unit Circle

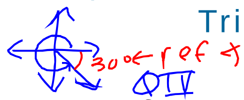
The diagram shows how the signs of the trigonometric functions depend on the quadrant containing the terminal side of θ in standard position.

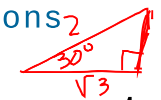
	S	A	
QII $(-,+)$	$\sin\theta: +$ $\cos\theta: -$ $\tan\theta: -$	$\sin\theta: +$ $\cos\theta: +$ $\tan\theta: +$	QI $(+,+)$
QIII $(-,-)$	$\sin\theta: -$ $\cos\theta: -$ $\tan\theta: +$	$\sin\theta: -$ $\cos\theta: +$ $\tan\theta: -$	QIV $(+,-)$
	T	C	

"All students take calculus."
"All students try cheating."

10-3 The Unit Circle

Example 2: Using Reference Angles to Evaluate Trigonometric functions





Use a reference angle to find the exact value of the sine, cosine, and tangent of 330°.

$\sin 330° = -\frac{1}{2}$

$\cos 330° = +\frac{\sqrt{3}}{2}$

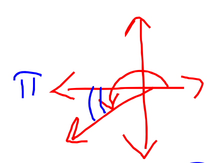
$\tan 330° = -\frac{\sqrt{3}}{3}$

10-3 The Unit Circle

Check It Out! Example 2b

Use a reference angle to find the exact value of the sine, cosine, and tangent of each angle.

$$\frac{4\pi}{3}$$



$\sin \frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$

ref ∠: $\frac{\pi}{3}$

$\cos \frac{4\pi}{3} = -\frac{1}{2}$

$\tan \frac{4\pi}{3} = +\sqrt{3}$