

PreCalculus
Midterm Exam Review

Name Key 2017-18
Date 1/19/24 Block 1A, 1B

Chapters 2/7

1. Find the following parts of the function: domain, x-intercept, y-intercept, vertical asymptote(s), horizontal asymptote(s), and/or slant asymptote(s). Domain can just show restrictions (also identify any holes), axis intercepts are points, and asymptotes are equations for x and y. State NONE if the value does not exist.

a.) $f(x) = \frac{x-3}{x^2 - 3x - 4} = \frac{(x-3)}{(x-4)(x+1)}$

Domain: $x \neq 4, -1$

x-intercept(s): (3, 0)

y-intercept(s): (0, 3/4)

H.A.: $y = 0$

V.A.: $x = 4, x = -1$

S.A.: N/A

Hole(s): none

b.) $f(x) = \frac{x^2 - 1}{x^2 - 2x - 3} = \frac{(x+1)(x-1)}{(x-3)(x+1)}$

Domain: $x \neq 3, -1$

x-intercept(s): (1, 0)

y-intercept(s): (0, 1/3)

H.A.: $y = 1$

V.A.: $x = 3$

S.A.: N/A

Hole(s): (-1, 1/2)

c.) $f(x) = \frac{x^2 - x - 2}{x - 1} = \frac{(x-2)(x+1)}{(x-1)}$

Domain: $x \neq 1$

x-intercept(s): (2, 0), (-1, 0)

y-intercept(s): (0, 2)

H.A.: N/A

V.A.: $x = 1$

S.A.: $y = x$

Hole(s): none

d.) $f(x) = \frac{2x+5}{x+1}$

Domain: $x \neq -1$

x-intercept(s): (-5/2, 0)

y-intercept(s): (0, 5)

H.A.: $y = 2$

V.A.: $x = -1$

S.A.: N/A

Hole(s): none

$$x-1 \sqrt{\frac{x-2/x-1}{x^2-x-2}} - \underline{x^2+x}$$

In 2 - 3, find the partial fraction decomposition of each.

$$2. \frac{-5x+4}{x^2-x} = -\frac{5x+4}{x(x-1)} = \left\{ \begin{array}{l} -\frac{4}{x} \\ -\frac{1}{x-1} \end{array} \right\}$$

$$3. \frac{-7x-15}{x^2+6x+9} = -\frac{7x-15}{(x+3)^2} = \left\{ \begin{array}{l} -\frac{7}{x+3} \\ +\frac{6}{(x+3)^2} \end{array} \right\}$$

$$\frac{A^{(x-1)}}{X(x-1)x(x-1)} + \frac{Bx}{(x-1)} = \frac{-5x+4}{x(x-1)}$$

$$Ax - A + Bx = -5x + 4$$

$$A + B = -5$$

$$B = -1$$

$$-A = 4$$

$$A = -4$$

$$\frac{A^{(x+3)}}{(x+3)(x+3)} + \frac{B}{(x+3)^2} =$$

$$Ax + 3A + B = -7x - 15$$

$$A = -7$$

$$3A + B = -15$$

$$-21 + B = -15$$

$$B = 6$$

Chapter 3

In 4 - 6, evaluate each expression WITHOUT A CALCULATOR.

$$4. \frac{\log_{12} 12^{36}}{\log_4 4^{18}} = \frac{36}{18} = 2$$

$$5. \ln e^{5a} = 5a$$

$$6. \log_4 320 - \log_4 5 = \log_4 \left(\frac{320}{5}\right) = \log_4 64 = 3$$

$$7. \text{ Use the change of base formula to evaluate: } \log_5 7 = \frac{\ln 7}{\ln 5} \approx 1.209$$

$$8. \text{ Use the properties of logarithms to expand: } \ln \frac{\sqrt{x^3 y^2}}{z} = \ln \frac{(x^3 y^2)^{1/2}}{z} =$$

$$9. \text{ Use the properties of logarithms to express the following expression as a single logarithm:}$$

$$\frac{1}{2}(\ln x^3 + \ln y^2) - \ln z = \frac{3}{2} \ln x + \ln y - \ln z$$

$$3 \ln(x-2) + 2 \ln(x+2) = \ln(x-2)^3 (x+2)^2$$

In 10 - 12, solve each equation algebraically. When necessary, round your result to the nearest thousandth.

$$10. 3^{2x} - 5 = 9$$

$$3^{2x} = 14$$

$$2 \times \log_3 3 = \log_3 14$$

$$2x = \frac{\ln 14}{\ln 3}$$

$$x = \left(\frac{\ln 14}{\ln 3} \right) \div 2 \approx 1.201$$

$$11. 3 + \log_2 3x = 5$$

$$\log_2 3x = 2$$

$$2^2 = 3x$$

$$x = \frac{4}{3}$$

$$12. \log(x) + \log(x-21) = 2$$

$$\log x(x-21) = 2$$

$$10^2 = x^2 - 21x$$

$$x^2 - 21x - 100 = 0$$

$$(x+4)(x-25) = 0$$

$$x = -4, 25$$

13. The number of bacteria present in culture $N(t)$ at time t hours is given by $N(t) = 3000(2)^t$.

a. What is the initial population?

3000 bacteria

b. How much bacteria are present after 24 hours? $N(24) = 3000(2)^{24} = 5.033 \times 10^{10}$ bacteria

c. How long will it take the population to triple in size?

$$9000 = 3000(2)^t$$

$$2^t = 3$$

$$t = \frac{\ln 3}{\ln 2} \approx 1.58 \text{ hours}$$

14. The number of students infected with flu after t days at Washington High School is modeled by the following function:

$$P(t) = \frac{1600}{1 + 99e^{-0.4t}}$$

a. What was the initial number of infected students? $P(0) = \frac{1600}{1+99} = \frac{1600}{100} = 16$ students

b. After 5 days, how many students will be infected?

$$P(5) = \frac{1600}{1 + 99e^{-0.4(5)}} = \frac{1600}{1 + 13.398} \approx 111.13 \text{ students}$$

c. What is the maximum number of students that will be infected?

#15. $y = \frac{3500}{1 + 34(0.346)^x}$

1600 students

15. The number of bacteria in a cup of water is modeled by a logistic curve. The limit to growth of the bacteria is 3500. The initial bacteria count is 100. After 3 hours, the bacteria count rises to 1450. Write the logistic function of the bacteria count.

$$y = \frac{c}{1 + ab^x} \rightarrow \frac{3500}{1+a} = 100 \rightarrow 1+a=35 \rightarrow a=34 \quad \frac{3500}{1+34b^3} = 1450 \rightarrow \frac{70}{29} = 1+34b^3$$

Chapter 4, Part I

16. Convert the angle measure from degrees to radians.

a.) $-270^\circ = \boxed{-\frac{3\pi}{2}}$

b.) $144^\circ = \boxed{\frac{4\pi}{5}}$

$$b^3 = \frac{41}{986}$$

$$b \approx 0.346$$

17. Convert the angle measure from radians to degrees.

a.) $\frac{7\pi}{3} = \boxed{420^\circ}$

b.) $\frac{-13\pi}{60} = \boxed{-39^\circ}$

18. a.) If the Earth rotates once every 24 hours, find the angular speed in radians/hour.

$$\frac{1 \text{ rotation}}{24 \text{ hours}} \cdot \frac{2\pi \text{ radians}}{1 \text{ rot.}} = \boxed{\frac{\pi}{12} \text{ radians/hour}} \approx 0.2618 \text{ radians/hr}$$

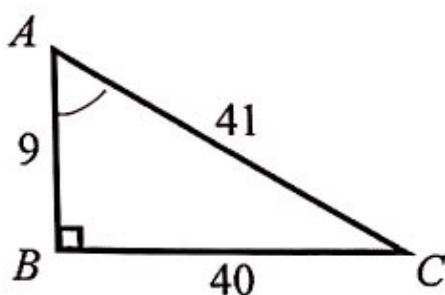
- b.) If a fan rotates 30 times in a minute, find the angular speed in radians/hour.

$$\frac{30 \text{ rotations}}{1 \text{ minute}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rot.}} = \boxed{3600\pi \text{ rad/hr}} \approx 11309.734 \text{ rad/hr}$$

- c.) If a ferris wheel rotates 4 times per minute, find the angular speed in radians/second.

$$\frac{4 \text{ rotations}}{1 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} \cdot \frac{2\pi \text{ radians}}{1 \text{ rot.}} = \boxed{\frac{2\pi}{15} \text{ rad/sec}} \approx 0.419 \text{ rad/sec}$$

19. Find the six trigonometric ratios of $\angle A$.



$$\sin A = \frac{40}{41}$$

$$\cos A = \frac{9}{41}$$

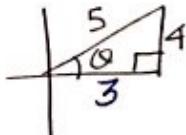
$$\tan A = \frac{40}{9}$$

$$\csc A = \frac{41}{9}$$

$$\sec A = \frac{41}{9}$$

$$\cot A = \frac{9}{40}$$

20. Given $\sin \theta = \frac{4}{5}$ in Quadrant I, find the remaining 5 trig ratios.



$$\cos \theta = \frac{3}{5}$$

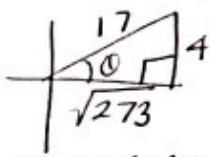
$$\tan \theta = \frac{4}{3}$$

$$\csc \theta = \frac{5}{4}$$

$$\sec \theta = \frac{5}{3}$$

$$\cot \theta = \frac{3}{4}$$

21. Given $\csc \theta = \frac{17}{4}$ in Quadrant I, find the remaining 5 trig ratios.



$$\sin \theta = \frac{4}{17}$$

$$\cos \theta = \frac{\sqrt{273}}{17}$$

$$\tan \theta = \frac{4\sqrt{273}}{273}$$

$$\sec \theta = \frac{17\sqrt{273}}{273}$$

$$\cot \theta = \frac{\sqrt{273}}{4}$$

22. Use a calculator to evaluate each function.

a.) $\sin 41^\circ$

0.6561

b.) $\cot 71.5^\circ$

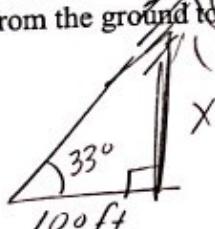
$\frac{1}{\tan 71.5^\circ} = 0.3346$

c.) $\cot \frac{\pi}{16}$

$\frac{1}{\tan \frac{\pi}{16}} = 5.0273$

d.) $\tan \frac{\pi}{8} = 0.4142$

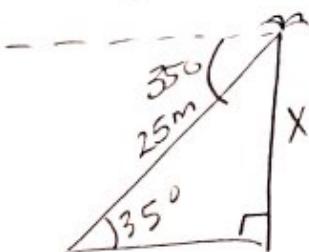
23. John wants to measure the height of a tree. He walks exactly 100 feet from the base of the tree and looks up. The angle from the ground to the top of the tree is 33° . How tall is the tree?



$$\tan 33^\circ = \frac{X}{100}$$

$$X = 100(\tan 33^\circ) = 64.94 \text{ feet}$$

24. A bird sits on top of a lamppost. The angle of depression from the bird to the feet of an observer standing away from the lamppost is 35° . The distance from the bird to the observer is 25 meters. How tall is the lamppost?



$$\sin 35^\circ = \frac{X}{25}$$

$$X = 25 \sin 35^\circ = 14.34 \text{ m}$$

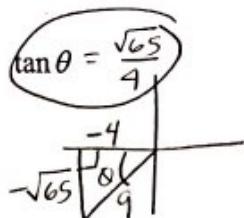
25. Determine two co-terminal angles (one positive and one negative) for each angle.

a.) $\theta = 52^\circ$ -308°
 412°

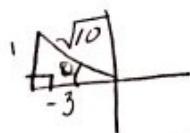
b.) $\theta = \frac{7\pi}{8}$ $-\frac{9\pi}{8}, \frac{23\pi}{8}$

26. Find the indicated trigonometric value in the specified quadrant.

a.) $\sec \theta = -\frac{9}{4}$; QIII; $\tan \theta = \frac{\sqrt{65}}{9}$



b.) $\cot \theta = -3$; QII; $\sin \theta = \frac{\sqrt{10}}{10}$



Chapter 4, Part II

27. Find the period and amplitude.

a.) $y = 3 \sin 2x$

amp = 3
 period = $\frac{2\pi}{2} = \pi$

b.) $y = \frac{2}{3} \sin \pi x$

amp = $\frac{2}{3}$
 per = $\frac{2\pi}{\pi} = 2$

c.) $y = \frac{3}{4} \cos \frac{\pi}{12} x$

amp = $\frac{3}{4}$
 period = $\frac{2\pi}{\pi/12} = 24$

28. Identify the transformation from f to g .

a.) $f(x) = \sin x$
 $g(x) = -4 \sin x$

• reflection in x -axis
 • vertical stretch by factor of 4

b.) $f(x) = \cos x$
 $g(x) = -\cos(x - \pi)$

• reflection in x -axis
 • horizontal shift right π

c.) $f(x) = 4 \sin \pi x$
 $g(x) = 4 \sin \pi x - 2$
 vertical shift down 2

29. Find the max and min.

a.) $y = 3 \sin x$

max: 3
 min: -3

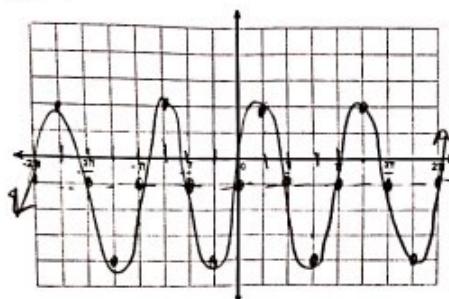
b.) $y = \frac{1}{2} \sin(x - \pi)$

max: $\frac{1}{2}$
 min: $-\frac{1}{2}$

30. Graph the following:

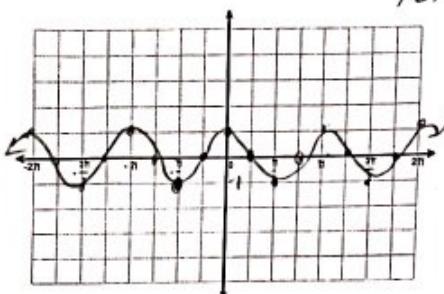
a.) $y = 3 \sin 2x - 1$

$\omega = 3$
period = π
down 1

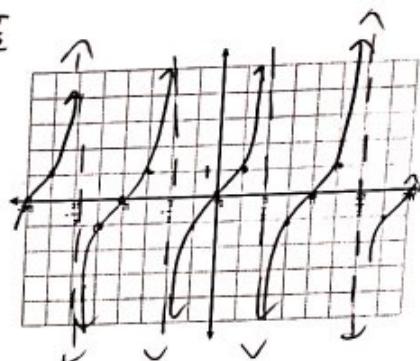


b.) $y = -\cos(2x + \pi) \quad \theta = 1$

period = π
phase shift left $\frac{\pi}{2}$



c.) $y = \tan x$

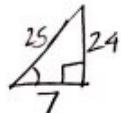


31. Find the exact value of the expression.

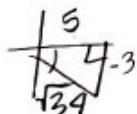
a.) $\sin(\arctan \frac{4}{3}) = \frac{4}{5}$



b.) $\cos(\arcsin \frac{24}{25}) = \frac{7}{25}$



c.) $\sec(\arctan(-\frac{3}{5})) = \frac{\sqrt{34}}{5}$



32. Find the exact value of y without a calculator.

a.) $y = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

b.) $y = \arctan(1) = \frac{\pi}{4}$

Chapter 5

33. Simplify the expression: $\cos \theta - \cos \theta \sin^2 \theta = \cos \theta (1 - \sin^2 \theta) = \cos \theta (\cos^2 \theta)$
 $= \underline{\cos^3 \theta}$

34. Simplify the expression: $\frac{\cos^2 x + \sin^2 x}{\cot^2 x - \csc^2 x} = \frac{1}{\cot^2 x - (\cot^2 x + 1)} = \frac{1}{-1} = -1$

35. Simplify the expression: $\cos x + \sin x \tan x = \cos x + \sin x \cdot \frac{\sin x}{\cos x}$

$$= \cos x + \frac{\sin^2 x}{\cos x} = \frac{\cos^2 x + \sin^2 x}{\cos x} = \frac{1}{\cos x} = \underline{\sec x}$$

36. Factor: $\sin^2 x + \sin x - 2$

$$\underline{(sin x + 2)(sin x - 1)}$$

37. Simplify the expression: $\frac{\sin^2 x - 1}{1 + \sin x} = \frac{(\sin x + 1)(\sin x - 1)}{1 + \sin x} = \underline{\sin x - 1}$