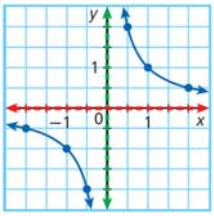
5-4 Rational Functions

A **rational function**

The parent rational function is $f(x) = \frac{1}{x}$. Its graph is a *hyperbola*, which has two separate branches. You will learn more about hyperbolas later in this course.

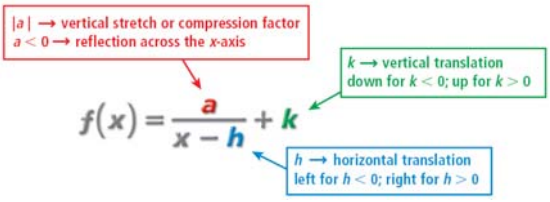
5-4 Rational Functions

The function $f(x) = \frac{1}{x}$ has a vertical asymptote at and a horizontal asymptote at



5-4 Rational Functions

The rational function $f(x) = \frac{1}{x}$ can be transformed by using methods similar to those used to transform other types of functions.



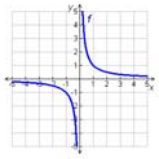
5-4 Rational Functions

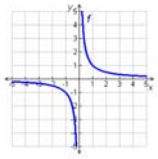
Example 1: Transforming Rational Functions

Using the graph of $f(x) = \frac{1}{x}$ as a guide, describe the transformation and graph each function.

A. $g(x) = \frac{1}{x + 2}$

B. $g(x) = \frac{1}{x} - 3$





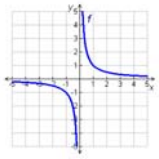
5-4 Rational Functions

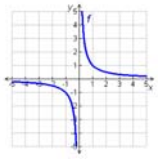
Check It Out! Example 1

Using the graph of $f(x) = \frac{1}{x}$ as a guide, describe the transformation and graph each function.

a. $g(x) = \frac{1}{x + 4}$

b. $g(x) = \frac{1}{x} + 1$





5-4 Rational Functions

The values of *h* and *k* affect the locations of the asymptotes, the domain, and the range of rational functions whose graphs are hyperbolas.

5-4 Rational Functions

Example 2: Determining Properties of Hyperbolas

Identify the asymptotes, domain, and range of the function $g(x) = \frac{1}{x + 3} - 2$.

Vertical asymptote:
Domain:
Horizontal asymptote:
Range:

Check Graph the function on a graphing calculator.

5-4 Rational Functions

Check It Out! Example 2

Identify the asymptotes, domain, and range of the function $g(x) = \frac{1}{x - 3} - 5$.

Vertical asymptote:
Domain:
Horizontal asymptote:
Range:

Check Graph the function on a graphing calculator.

5-4 Rational Functions

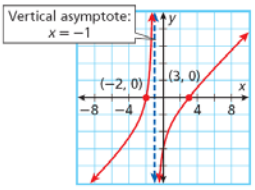
A **discontinuous function**

A **continuous function**

5-4 Rational Functions

The graphs of some rational functions are not hyperbolas. Consider the rational function $f(x) = \frac{(x - 3)(x + 2)}{x + 1}$ and its graph.

The numerator of this function is 0 when $x = 3$ or $x = -2$. Therefore, the function has x-intercepts at -2 and 3. The denominator of this function is 0 when $x = -1$. As a result, the graph of the function has a vertical asymptote at the line $x = -1$.



5-4 Rational Functions

5-4 Rational Functions

Example 3: Graphing Rational Functions with Vertical Asymptotes

Identify the zeros and vertical asymptotes of $f(x) = \frac{(x^2 + 3x - 4)}{x + 3}$.

Step 1 Find the zeros and vertical asymptotes.

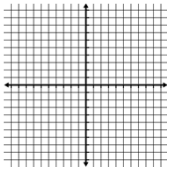
5-4 Rational Functions

Example 3 Continued

Identify the zeros and vertical asymptotes of $f(x) = \frac{(x^2 + 3x - 4)}{x + 3}$.

Step 2 Graph the function.

Plot the zeros and draw the asymptote. Then make a table of values to fill in missing points.



x							
y							

5-4 Rational Functions

Check It Out! Example 3

Identify the zeros and vertical asymptotes of $f(x) = \frac{(x^2 + 7x + 6)}{x + 3}$.

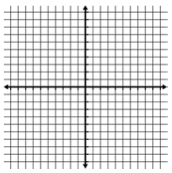
Step 1 Find the zeros and vertical asymptotes.

5-4 Rational Functions

Check It Out! Example 3 Continued

Identify the zeros and vertical asymptotes of $f(x) = \frac{(x^2 + 7x + 6)}{x + 3}$.

Step 2 Graph the function.



x							
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5-4 Rational Functions