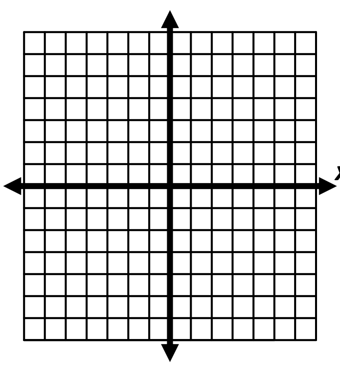
Algebra 2 Honors **Name____________________**
Notes/Exploration: 3-7, 3-8 **Date___________Block_____**

Investigating Graphs of Polynomial Functions
Transforming Polynomial Functions

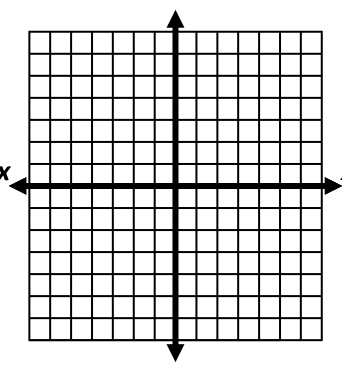
- Polynomial functions are classified by their degree.
- The graphs of polynomial functions are classified by the degree of the polynomial.
- Each graph, based on the degree, has a distinctive shape and characteristics.
- **End behavior is a description of the values of the function as x approaches infinity $(x \to \infty)$ or negative infinity $(x \to -\infty)$.**
- It is helpful when you are graphing a polynomial function to know about the end behavior of the function.

Sketch the following functions and describe the end behavior of each function.

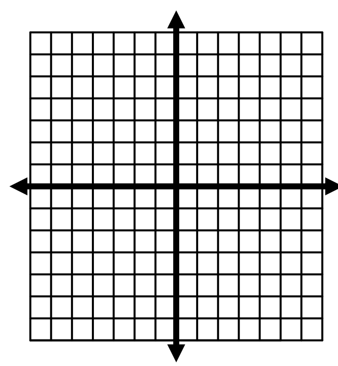
$f(x) = x^2$



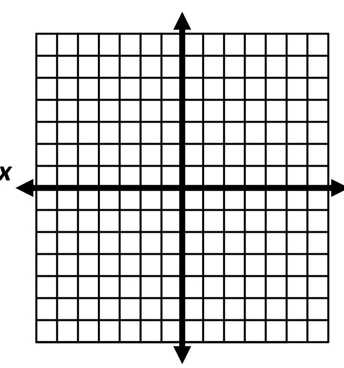
$f(x) = -x^2$



$f(x) = x^3$



$f(x) = -x^3$



End Behavior			
as $x \to \infty$, $f(x) \to$	as $x \to \infty$, $f(x) \to$	as $x \to \infty$, $f(x) \to$	as $x \to \infty$, $f(x) \to$
as $x \to -\infty$, $f(x) \to$	as $x \to -\infty$, $f(x) \to$	as $x \to -\infty$, $f(x) \to$	as $x \to -\infty$, $f(x) \to$

Now explore the graphs of some other polynomial functions on your own, and make a conjecture about the characteristics of the function that seem to affect its end behavior. Write your thoughts/conjecture here, and then we'll summarize our findings together.

Function	End Behavior	

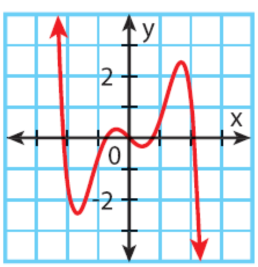
Example 1: Determining End Behavior of Polynomial Functions
Identify the leading coefficient, degree, and end behavior.

A. $Q(x) = -x^4 + 6x^3 - x + 9$

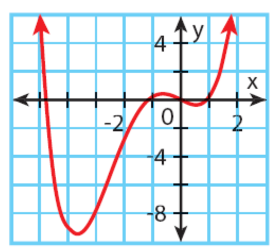
B. $P(x) = 2x^5 + 6x^4 - x + 4$

Example 2: Using Graphs to Analyze Polynomial Functions
Identify whether the function graphed has an odd or even degree and a positive or negative leading coefficient.

A.



B.

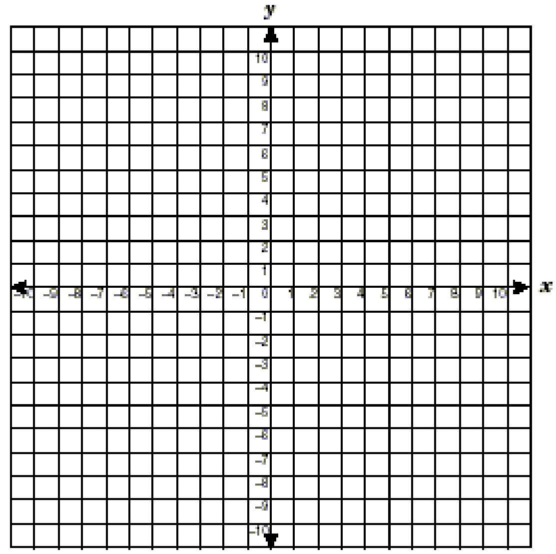


Steps for Graphing a Polynomial Function
1. Find the real zeros and *y*-intercept of the function.
2. Plot the *x*- and *y*-intercepts.
3. Make a table for several *x*-values that lie between the real zeros.
4. Plot the points from your table.
5. Determine the end behavior of the graph.
6. Sketch the graph.

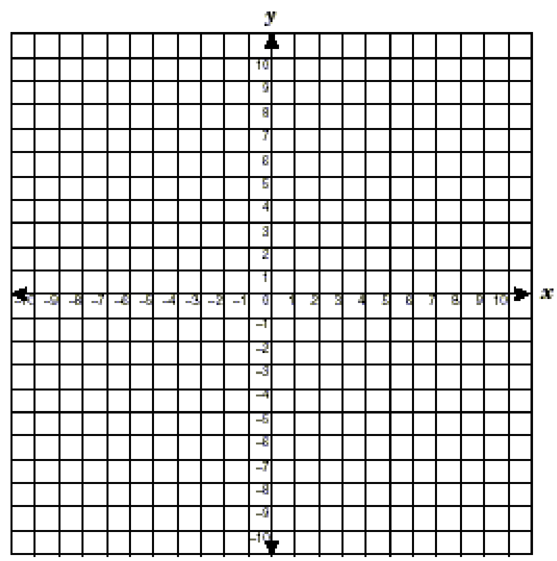
Example 3: Graphing Polynomial Functions

Graph each function.

A. $f(x) = x^3 + 4x^2 + x - 6$



B. $f(x) = -x^3 + 2x^2 + 5x - 6$



- A **<u>turning point</u>** is where a graph changes from increasing to decreasing or from decreasing to increasing. A turning point corresponds to a *local maximum* or *minimum*.
- A polynomial function of degree *n* has ***at most*** *n* – 1 turning points and ***at most*** *n* *x*-intercepts.
- You can use a graphing calculator to graph and estimate maximum and minimum values.

Local Maxima and Minima

For a function $f(x)$, $f(a)$ is a **local maximum** if there is an interval around a such that $f(x) < f(a)$ for every x-value in the interval except a.

For a function $f(x)$, $f(a)$ is a **local minimum** if there is an interval around a such that $f(x) > f(a)$ for every x-value in the interval except a.

Example 4: Determine Maxima and Minima with a Calculator

A. Graph $f(x) = 2x^3 - 18x + 1$ on a calculator, and estimate the local maxima and minima.

B. Graph $f(x) = x^3 - 2x - 3$ on a calculator, and estimate the local maxima and minima.

Example 5: Translating a Polynomial Function

For $f(x) = x^3 - 6$, write the rule for each function and identify the transformation.

A. $g(x) = f(x) - 2$

B. $h(x) = f(x + 3)$

Example 6:Reflecting a Polynomial Function

For $f(x) = x^3 + 5x^2 - 8x + 1$, write the rule for each function and identify the transformation.

A. $h(x) = -f(x)$

B. $g(x) = f(-x)$

Example 7: Compressing and Stretching a Polynomial Function

For $f(x) = 2x^4 - 6x^2 + 1$, write the rule for each function and identify the transformation.

A. $g(x) = \frac{1}{2} f(x)$

B. $g(x) = f\left(\frac{1}{3}x\right)$

Example 8: Combining Transformations

Write a function that transforms $f(x) = 6x^3 - 3$ in each of the following ways. Support your answer by using a graphing calculator.

A. Compress vertically by a factor of $\frac{1}{2}$, and shift 3 units right.

B. Reflect across the *y*-axis and shift 2 units down.