

Part I

The Fundamental Theorem of Algebra

Every polynomial function of degree $n \geq 1$ has at least one zero, where a zero may be a complex number

Corollary: Every polynomial function of degree $n \geq 1$ has _____
_____.

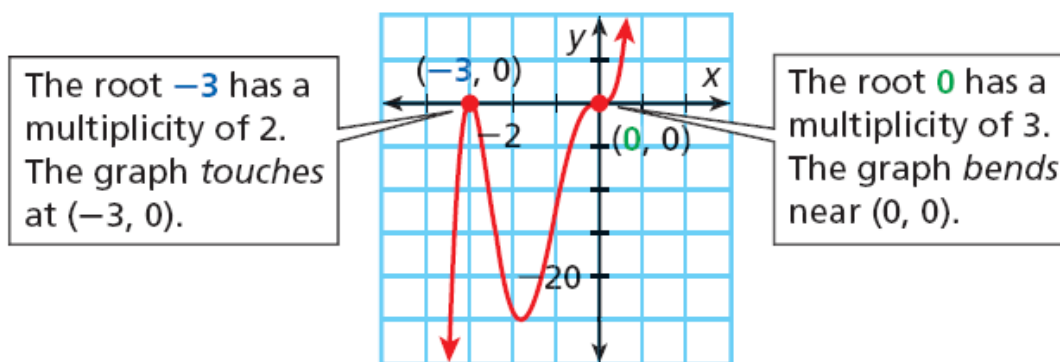
Examples: Solve each polynomial equation by factoring.

1. $x^3 - 2x^2 - 25x = -50$

2. $4x^6 + 4x^5 - 24x^4$

3. $x^4 + 25 = 26x^2$

- The **multiplicity** of root r is the _____.
- When a real root has even multiplicity, the graph of $y = P(x)$ _____.
- When a real root has odd multiplicity greater than 1, the graph _____.



Examples: Identify the roots of each equation. State the multiplicity of each root.

4. $x^3 + 6x^2 + 12x + 8 = 0$

5. $x^4 + 8x^3 + 18x^2 - 27 = 0$

For You ☺

Examples: Solve each equation by factoring. State the multiplicity of each root.

6. $x^3 + 6x^2 - 5x - 30 = 0$

7. $2x^5 + 12x^4 + 16x^3 - 12x^2 - 18x = 0$

Part II

Rational Root Theorem

If the polynomial $P(x)$ has integer coefficients, then every rational root of the polynomial equation $P(x) = 0$ can

be written in the form _____
_____.

Irrational Root Theorem

If the polynomial $P(x)$ has rational coefficients and _____ is a root of the polynomial equation

$P(x) = 0$, where a and b are rational and \sqrt{c} is irrational, then _____.

Complex Conjugate Root Theorem

If _____ is a root of a polynomial equation with real-number coefficients, then _____
is also a root.

Examples: Solve each equation by finding all roots.

1. $4x^4 - 21x^3 + 18x^2 + 19x - 6 = 9$

2. $x^4 + x^3 + 2x^2 + 4x - 8 = 0$

3. $2x^3 - 9x^2 + 2 = 0$

4. $x^4 - 3x^3 + 5x^2 - 27x - 36 = 0$

5. Write the simplest polynomial with roots $-1, \frac{2}{3},$ and $4.$

6. Write the simplest polynomial function with the given zeros: $0, -4, \sqrt{3}$

7. Write the simplest polynomial function with zeros $2i, 1 + \sqrt{2}$, and 3

Wrap Up

Write the simplest polynomial function with the given zeros.

8. 2, -1, 1

9. 0, -2, $\sqrt{3}$

10. $2i$, 1, -2

11. Solve by finding all roots: $x^4 - 5x^2 + 7x^2 - 5x + 6 = 0$