2017-18

PreCalculus
Final Exam Review

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Date______ Block______

Chapter 5 (Sections 5.3 – 5.5)

1. Find all solutions for the variable in the interval $[0, 2\pi)$.

a.) $2\sin^2 x + 3\cos x - 3 = 0$

$2(1-\cos^2 x) + 3\cos x - 3 = 0$

$-2\cos^2 x + 3\cos x - 1 = 0$

$2\cos^2 x - 3\cos x + 1 = 0$

$(2\cos x - 1)(\cos x - 1) = 0$

$\cos x = 1/2$

$\cos x = 1$

$\boxed{x = \left\{\frac{\pi}{3}, \frac{5\pi}{3}, 0\right\}}$

b.) $2\cos^2 x = \cos x$

$2\cos^2 x - \cos x = 0$

$\cos x(2\cos x - 1) = 0$

$\cos x = 0 \quad \cos x = 1/2$

$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{3}, \frac{5\pi}{3}}$

c.) $3\tan^3 x - \tan x = 0$

$\tan x(3\tan^2 x - 1) = 0$

$\tan x = 0 \quad \tan^2 x = 1/3$

$\tan x = \frac{\pm\sqrt{3}}{3}$

$x = 0, \pi$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

d.) $\sin 2x - \cos x = 0$

$2\sin x\cos x - \cos x = 0$

$\cos x(2\sin x - 1) = 0$

$\cos x = 0 \quad 2\sin x - 1 = 0$

$\sin x = 1/2$

$\boxed{x = \pi/2, 3\pi/2, \pi/6, 5\pi/6}$

2. Use half-angle formulas to find the exact value.

a.) $\cos\left(\frac{\pi}{8}\right) = \cos\left(\frac{\pi/4}{2}\right) = \sqrt{\frac{1+\cos\pi/4}{2}} = \sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{2+\sqrt{2}}{4}} = \frac{\sqrt{2+\sqrt{2}}}{2}$

b.) $\tan\left(\frac{3\pi}{8}\right) = \tan\left(\frac{3\pi/4}{2}\right) = \frac{1-\cos\frac{3\pi}{4}}{\sin\frac{3\pi}{4}} = \frac{1+\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \frac{\frac{2+\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \frac{2+\sqrt{2}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{2\sqrt{2}+2}{2} = \boxed{\sqrt{2}+1}$

c.) $\sin\left(\frac{\pi}{12}\right) = \sin\frac{\pi/6}{12} = \sqrt{\frac{1-\cos\frac{\pi}{6}}{2}} = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2-\sqrt{3}}{4}} = \frac{\sqrt{2-\sqrt{3}}}{2}$

3. Write the expression as the sine, cosine, or tangent of an angle.

a.) $\cos 60^\circ \cos 10^\circ - \sin 60^\circ \sin 10^\circ = \cos(60^\circ + 10^\circ) = \cos 70^\circ$

b.) $\frac{\tan 152^\circ - \tan 47^\circ}{1 + \tan 152^\circ \tan 47^\circ} = \tan(152^\circ - 47^\circ) = \tan 105^\circ$

c.) $\sin\frac{4\pi}{9}\cos\frac{\pi}{8} + \cos\frac{4\pi}{9}\sin\frac{\pi}{8} = \sin\left(\frac{4\pi}{9} + \frac{\pi}{8}\right) = \sin\frac{41\pi}{72}$

4. Verify each identity.

a.) $\cos\left(\theta+\frac{\pi}{2}\right)-\cos\left(\theta-\frac{\pi}{2}\right)=-2\sin\theta$

$(\cos\theta\cos\frac{\pi}{2} - \sin\theta\sin\frac{\pi}{2}) - (\cos\theta\cos\frac{\pi}{2} + \sin\theta\sin\frac{\pi}{2}) =$

$-2\sin\theta\sin\frac{\pi}{2} =$

$-2\sin\theta(1) =$

$-2\sin\theta = -2\sin\theta$ ✓

b.) $\sec 2x = \frac{\sec^2 x}{2-\sec^2 x}$

See attached

c.) $\frac{\cos x + \cos 3x}{\sin 3x - \sin x} = \cot x$

See attached

d.) $(\sin x + \cos x)^2 = 1 + \sin 2x$

$(\sin x + \cos x)(\sin x + \cos x) =$

$\sin^2 x + 2\sin x\cos x + \cos^2 x =$

$1 + 2\sin x\cos x =$

$1 + \sin 2x = 1 + \sin 2x$ ✓

Chapter 6

In 5 - 7, solve the triangle for all angles and sides. If two solutions exist, find both.

5. $c = 13, b = 8, B = 31°$

$\frac{8}{\sin 31°} = \frac{13}{\sin C}$

$\sin C = \frac{13\sin 31°}{8}$

$\frac{a}{\sin 92.18°} = \frac{8}{\sin 31°}$

$a = \frac{8\sin 92.18°}{\sin 31°}$

$C = 56.82°$ $A = 92.18°$ $a = 15.52$	OR	$C = 123.18°$ $A = 25.82°$ $a = 6.77$

6. $A = 55°, b = 12, c = 7$

$a^2 = 12^2 + 7^2 - 2(12)(7)\cos 55°$

$\boxed{a = 9.83}$

$\frac{9.83}{\sin 55°} = \frac{12}{\sin B}$

$\boxed{B \approx 89.66°}$

$\boxed{C \approx 35.34°}$

7. $A = 33°, B = 70°, b = 7$

$C = 77°$

$\frac{7}{\sin 70°} = \frac{a}{\sin 33°}$

$a = \frac{7\sin 33°}{\sin 70°} = \boxed{4.06}$

$\frac{7}{\sin 70°} = \frac{c}{\sin 77°}$

$c = \frac{7\sin 77°}{\sin 70°} = 7.26$

In 8 - 9, find the area of the triangle to the nearest tenth.

8. $A = 52°, b = 14$ m, $c = 21$ m

$Area = \frac{1}{2}bc\sin A$

$Area = \frac{1}{2}(14)(21)\sin 52° \approx \boxed{115.84\,m^2}$

9. $a = 7$ cm, $b = 8$ cm, $c = 9$ cm

$S = 12$

$Area = \sqrt{12(12-7)(12-8)(12-9)} = \sqrt{720} \approx \boxed{26.83\,cm^2}$

4b) $\sec 2x = \dfrac{\sec^2 x}{2-\sec^2 x}$

$\dfrac{1}{\cos 2x} =$

$\dfrac{1}{2\cos^2 x - 1} =$

$\dfrac{1}{2\left(\frac{1}{\sec^2 x}\right) - 1} =$

$\dfrac{1}{\frac{2-\sec^2 x}{\sec^2 x}} =$

$\dfrac{\sec^2 x}{2-\sec^2 x} = \dfrac{\sec^2 x}{2-\sec^2 x}$ ✓

4c) $\dfrac{\cos x + \cos 3x}{\sin 3x - \sin x} = \cot x$

$\dfrac{2\cos\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}{2\cos\left(\frac{3x+x}{2}\right)\sin\left(\frac{3x-x}{2}\right)} =$

$\dfrac{\cancel{2\cos 2x}\cos(-x)}{\cancel{2\cos 2x}\sin x}$

$\dfrac{\cos x}{\sin x} =$

$\cot x = \cot x$ ✓

Chapter 9

In 10 - 12, write the equation in standard form and then classify the graph as a parabola, circle, ellipse, or hyperbola.

CIRCLE 10. $x^2+y^2-6x+4y+9=0$

$(x^2-6x+9)+(y^2+4y+4)=-9+9+4$

$\boxed{(x-3)^2+(y+2)^2=4}$

ELLIPSE 11. $x^2-6x+16y+21=-4y^2$

$(x^2-6x+9)+4(y^2+4y+4)=-21+9+16$

$\dfrac{(x-3)^2+4(y+2)^2=4}{4}$

$\boxed{\dfrac{(x-3)^2}{4}+\dfrac{(y+2)^2}{1}=1}$

PARABOLA 12. $y^2-6y-4x+21=0$

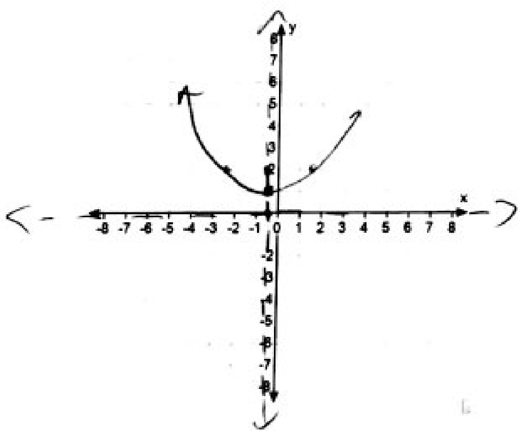
$y^2-6y+9=4x-21+9$

$(y-3)^2=4x-12$

$\boxed{(y-3)^2=4(x-3)}$

13. Find the vertex, axis of symmetry, focus, and directrix of the parabola and sketch its graph.

$$\left(x+\frac{1}{2}\right)^2=4(y-1)$$



$p=1$
opens up

Vertex: $(-1/2, 1)$

Focus: $(-1/2, 2)$

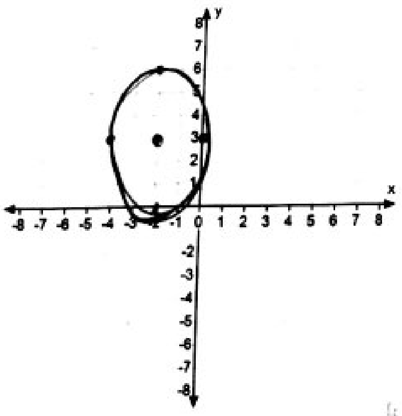
directrix $y=0$

AOS: $x=-1/2$

14. Identify the conic as a circle or ellipse. Then find the center and radius (if it's a circle); find the center, vertices, co-vertices, and foci (if it's an ellipse). Sketch its graph.

$$9x^2+4y^2+36x-24y+36=0$$

Ellipse



$9(x^2+4x+4)+4(y^2-6y+9)=-36+36+36$

$\dfrac{9(x+2)^2+4(y-3)^2=36}{36}$

$\dfrac{(x+2)^2}{4}+\dfrac{(y-3)^2}{9}=1$

$c^2=a^2-b^2$
$c^2=9-4$
$c=\sqrt{5}$

Center: $(-2, 3)$

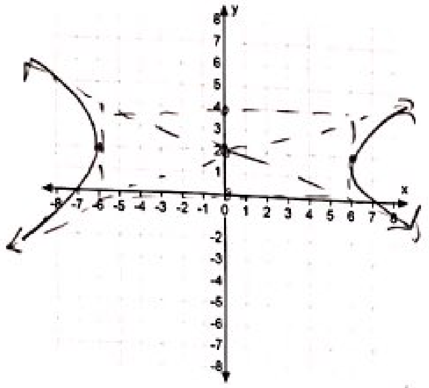
Vertices: $(-2, 6), (-2, 0)$

Co-vertices. $(-4, 3), (0, 3)$ Foci. $(-2, 3\pm\sqrt{5}$

$(0,2)$ $(-6,2)(6,2)$ Conjugate: $x=0$ Transverse: $y=2$ Asymptotes: $2 \pm \frac{1}{3}x$

15. Find the center, vertices, foci, lines containing the axes, and the equations of the asymptotes of the hyperbola, and then sketch its graph.

$$x^2 - 9y^2 + 36y - 72 = 0$$



$x^2 - 9(y^2 - 4y + 4) = 72 - 36$

$\dfrac{x^2 - 9(y-2)^2 = 36}{36}$

$\frac{x^2}{36} - \frac{(y-2)^2}{4} = 1$ Foci: $(\pm 2\sqrt{10}, 2)$

$c^2 = a^2 + b^2 = 36 + 40$ $c = 2\sqrt{10}$

16. Write the equation of a circle that has a center at (-1, 3) and passes through the point (-5, 6).

$r = \sqrt{(-1+5)^2 + (3-6)^2}$

$r = \sqrt{4^2 + (-3)^2} = \sqrt{25} = 5$

$\boxed{(x+1)^2 + (y-3)^2 = 25}$

Sequences and Series

In 17 - 18, write the explicit formula for each sequence.

17. -3, -6, -12, -24, -48, …

$\boxed{a_n = -3(2)^{n-1}}$

18. -4, -14, -24, -34, -44, …

$\boxed{a_n = -4 - 10(n-1)}$

19. Find "n" if you know that $S_n = 59{,}046$ in the series $6 + 18 + 54 + 162\ldots$

$n = 9$

$59046 = 6\left(\frac{1-3^n}{1-3}\right)$

$a_n = 6 \cdot 3^{n-1}$

$-19682 = 1 - 3^n$

$19683 = 3^n$

$n = \log_3 19683$

20. Evaluate $\sum_{n=0}^{5}(20 - n^2) = 20 + 19 + 16 + 11 + 4 + (-5) = 65$

21. Evaluate $\binom{12}{3} = \frac{12!}{3!9!} = 220$

In 22 - 23, find each term described.

22. 2nd term in expansion of $(x+3)^3$

$\binom{3}{1}(x)^2(3)^1 = 9x^2$

23. 4th term in expansion of $(3u-1)^4$

$\binom{4}{3}(3u)^1(-1)^3 = -12u$

In 24 - 25 expand completely.

24. $(2y - x)^4$

$\boxed{16y^4 - 32y^3x + 24y^2x^2 - 8yx^3 + x^4}$

25. $(2y + 3x)^3$

$\boxed{8y^3 + 36y^2x + 54yx^2 + 27x^3}$