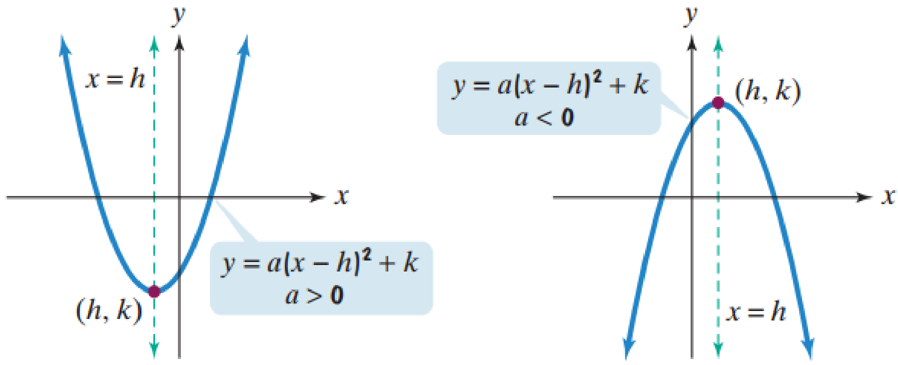
Parabolas: Quick Review

Here's a brief summary:

Graphing $y = a(x - h)^2 + k$ and $y = ax^2 + bx + c$

1. If $a > 0$, the graph opens upward. If $a < 0$, the graph opens downward.
2. The vertex of $y = a(x - h)^2 + k$ is (h, k).



3. The x-coordinate of the vertex of $y = ax^2 + bx + c$ is $x = -\frac{b}{2a}$.

Definition of a Parabola

A **parabola** is the set of all points in a plane that are equidistant from a fixed line, the **directrix**, and a fixed point, the **focus**, that is not on the line (see **Figure 9.29**).

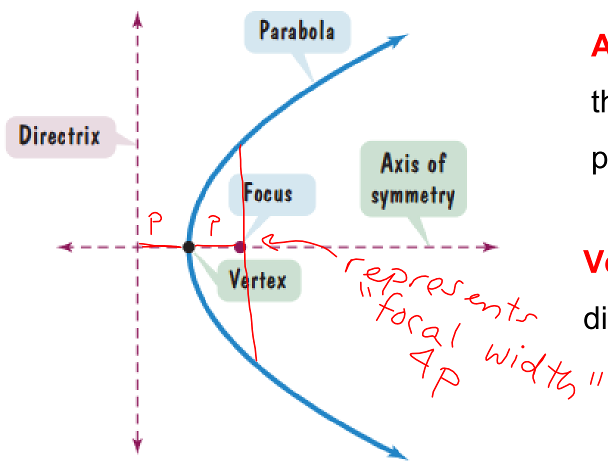


FIGURE 9.29

Axis of Symmetry: the line passing through the focus and vertex, and is perpendicular to the directrix.

Vertex: the midpoint of the focus and directrix.

Standard Equation of a Parabola

The **standard form of the equation of a parabola** with vertex at (h, k) is as follows.

$(x-h)^2 = 4p(y-k),\ p \neq 0$ — Vertical axis, directrix: $y = k - p$

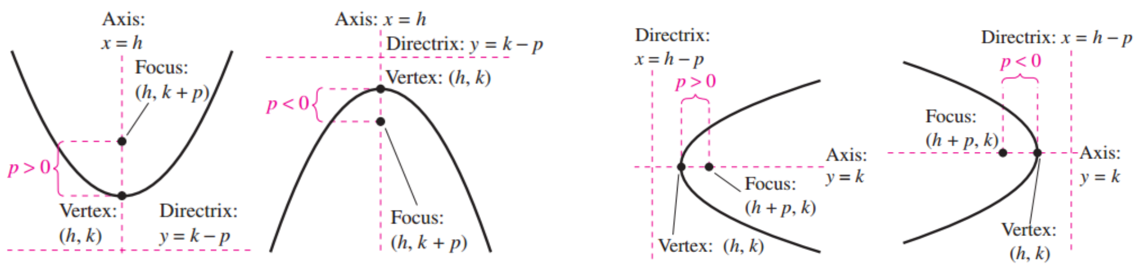
$(y-k)^2 = 4p(x-h),\ p \neq 0$ — Horizontal axis, directrix: $x = h - p$

The focus lies on the axis p units (*directed distance*) from the vertex. If the vertex is at the origin $(0, 0)$, the equation takes one of the following forms.

$x^2 = 4py$ — Vertical axis

$y^2 = 4px$ — Horizontal axis

See Figure 10.11.



(a) $(x-h)^2 = 4p(y-k)$ Vertical axis: $p > 0$
(b) $(x-h)^2 = 4p(y-k)$ Vertical axis: $p < 0$
(c) $(y-k)^2 = 4p(x-h)$ Horizontal axis: $p > 0$
(d) $(y-k)^2 = 4p(x-h)$ Horizontal axis: $p < 0$

FIGURE 10.11

1.) Find the vertex, focus, and directrix of the parabola given by:

$$(x-3)^2 = \overset{4p}{8}(y+1)$$

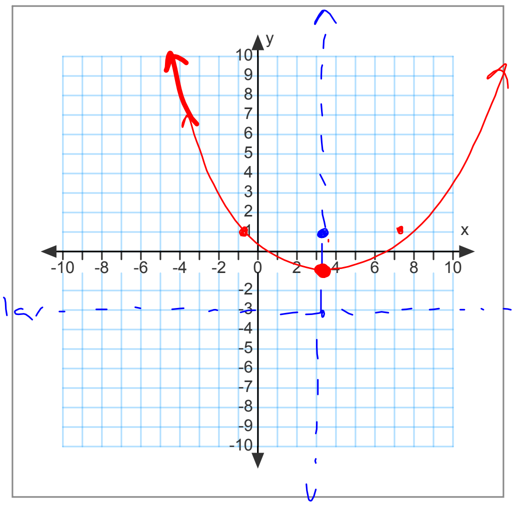
$p = 2$

$(x-h)^2 = 4p(y-k),\ p \neq 0$

$(y-k)^2 = 4p(x-h),\ p \neq 0$

focal width

Then graph the parabola.



(h, k)

Coordinate of Vertex: (3, -1)

Direction it Opens: up

Axis of Symmetry: x = 3

Coordinate of Focus: (3, 1)

always in the interior of parabola

Equation of Directrix: y = -3

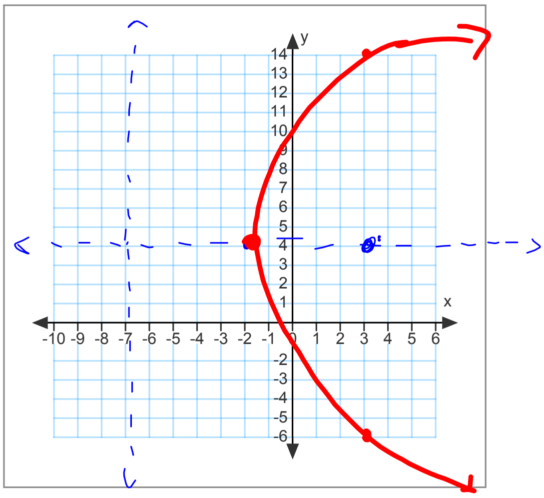
2.) Find the vertex, focus, and directrix of the parabola given by:

$$(y-4)^2 = 20(x+2)$$

$(x-h)^2 = 4p(y-k),\ p \neq 0$

$(y-k)^2 = 4p(x-h),\ p \neq 0$

Then graph the parabola. $p=5$



Coordinate of Vertex: (-2, 4)

Direction it Opens: right

Axis of Symmetry: y = 4

Coordinate of Focus: (3, 4)

Equation of Directrix: x = -7

3.) Find the vertex, focus, and directrix of the parabola given by:

$$y^2 + 2y + 12x - 23 = 0$$

$(x-h)^2 = 4p(y-k),\ p \neq 0$

$(y-k)^2 = 4p(x-h),\ p \neq 0$

Then graph the parabola. $y^2+2y+1 = -12x+23+1$

$(y+1)^2 = -12x+24$

$(y+1)^2 = -12(x-2)$

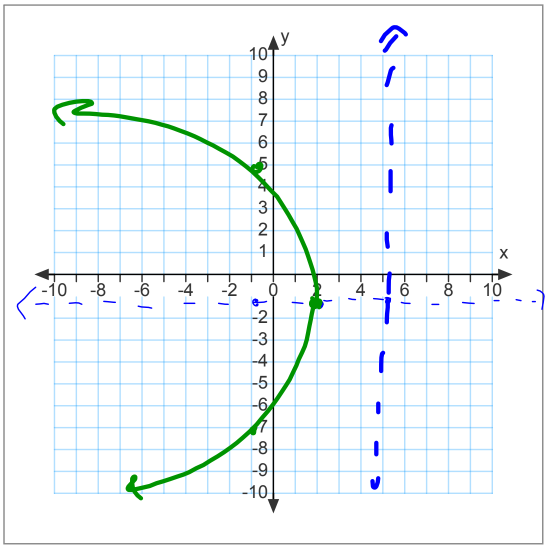
Coordinate of Vertex: (2, -1)

Direction it Opens: left

Axis of Symmetry: y = -1

Coordinate of Focus: (-1, -1)

Equation of Directrix: x = 5



4.) Find the vertex, focus, and directrix of the parabola given by:

$$y^2 + 21 = -20x - 6y - 68$$

$(x-h)^2 = 4p(y-k),\ p \neq 0$

$(y-k)^2 = 4p(x-h),\ p \neq 0$

Then graph the parabola. $y^2 + 6y + 9 = -20x - 89 + 9$

$(y+3)^2 = -20(x+4)$

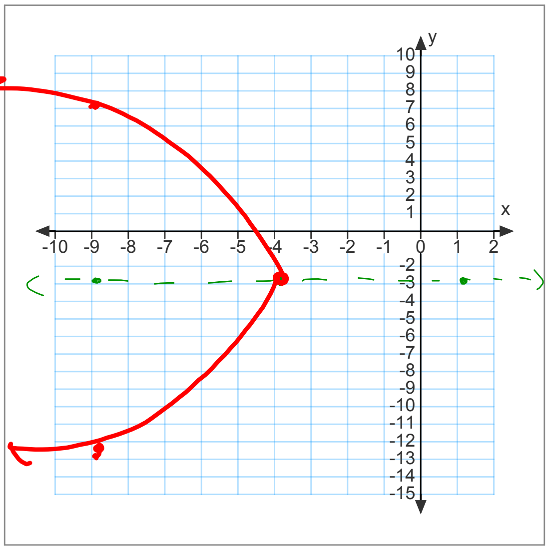
Coordinate of Vertex: $(-4,-3)$

Direction it Opens: left

Axis of Symmetry: $y=-3$

Coordinate of Focus: $(-9,-3)$

Equation of Directrix: $x=1$



5.) Find the vertex, focus, and directrix of the parabola given by:

$$y = -\frac{1}{2}x^2 - x + \frac{1}{2}.$$

$(x-h)^2 = 4p(y-k),\ p \neq 0$

$(y-k)^2 = 4p(x-h),\ p \neq 0$

Then graph the parabola.

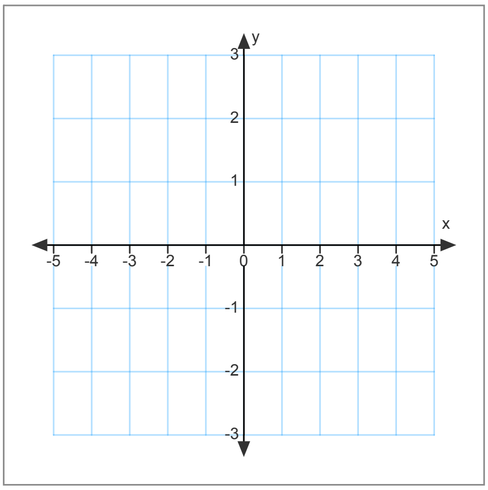
Coordinate of Vertex:

Direction it Opens:

Axis of Symmetry:

Coordinate of Focus:

Equation of Directrix:



6.) Find the standard form of a parabola with vertex at the origin and focus (0, 4).

(h, k)
$(0, 0)$

$(x-h)^2 = 4p(y-k)$

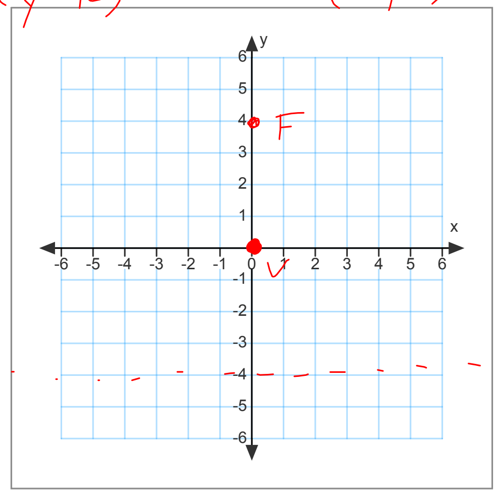
directrix : $y = -4$

$p = 4$

$4p = 16$

opens up

$\boxed{x^2 = 16y}$



7.) Find the standard form of a parabola with vertex $(1, 0)$ and focus $(2, 0)$.

(h, k)

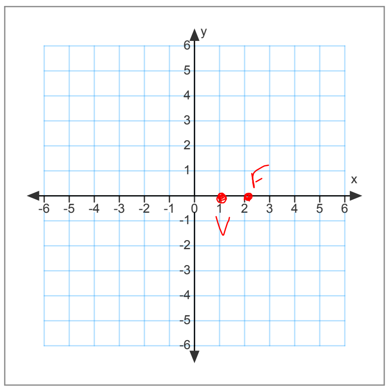
$(y-k)^2 = 4p(x-h)$

opens right

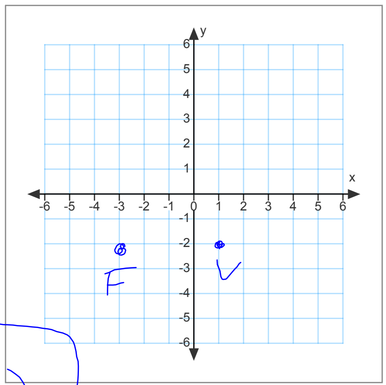
$p = 1$

$4p = 4$

$\boxed{y^2 = 4(x-1)}$



8.) Find the standard form of a parabola with vertex at (1, -2) and focus (-3, -2).

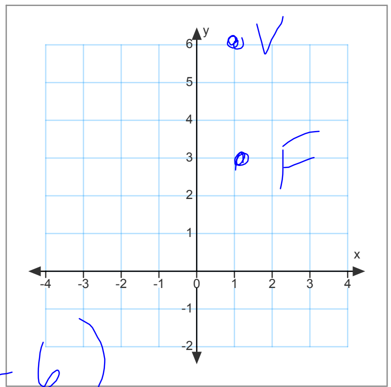


opens left

$p = -4$

$$\boxed{(y+2)^2 = -16(x-1)}$$

9.) Find the standard form of a parabola with vertex at (1, 6) and focus (1, 3).



opens down

$p = -3$

$$(x-1)^2 = -12(y-6)$$

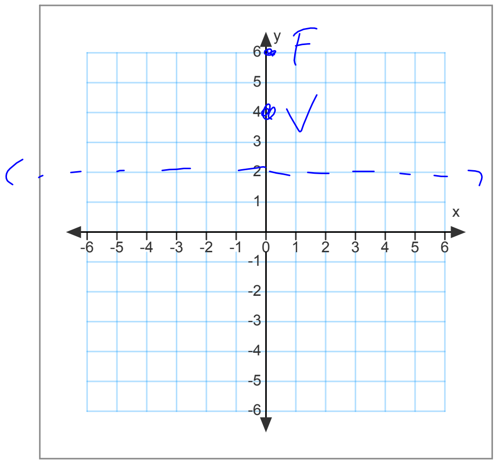
10.) Find the standard form of the equation of the parabola if the Vertex is (0, 4) and the directrix is y = 2.

(h, k)

p = 2

opens up

$x^2 = 8(y-4)$



Applications: The parabolic arch shown in the figure is 50 feet above the water at the center and 200 feet wide at the base. Will a boat that is 30 feet tall clear the arch 30 feet from the center?