2017

PreCalculus
WS: 5.4 – 5.5 Review

Name: Key
Date: 1/12, 1/13 Block: 1A, 2B

Find the exact value.

1.) $\cos \overset{u}{25^\circ} \cos \overset{v}{20^\circ} - \sin 25^\circ \sin 20^\circ$

$\cos(\overset{u}{25^\circ} + \overset{v}{20^\circ}) = \cos 45^\circ = \frac{\sqrt{2}}{2}$

2.) $\sin\frac{\pi}{16}\cos\frac{3\pi}{16} + \cos\frac{\pi}{16}\sin\frac{3\pi}{16}$

$\sin\left(\frac{\pi}{16} + \frac{3\pi}{16}\right) = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

Find the exact value of the sine, cosine, and tangent of the angle by using the sum and difference formulas.

3.) $\frac{5\pi}{12}$ Hint: $\frac{5\pi}{12} = \overset{u}{\frac{\pi}{6}} + \overset{v}{\frac{\pi}{4}}$

$\sin\left(\frac{5\pi}{12}\right) = \sin\left(\frac{\pi}{6} + \frac{\pi}{4}\right) = \sin\frac{\pi}{6}\cos\frac{\pi}{4} + \cos\frac{\pi}{6}\sin\frac{\pi}{4} = \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}+\sqrt{6}}{4}$

$\cos\left(\frac{5\pi}{12}\right) = \cos\left(\frac{\pi}{6} + \frac{\pi}{4}\right) = \cos\frac{\pi}{6}\cos\frac{\pi}{4} - \sin\frac{\pi}{6}\sin\frac{\pi}{4} = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{6}-\sqrt{2}}{4}$

$\tan\frac{5\pi}{12} = \tan\left(\frac{\pi}{6} + \frac{\pi}{4}\right) = \frac{\tan\frac{\pi}{6} + \tan\frac{\pi}{4}}{1 - \tan\frac{\pi}{6}\tan\frac{\pi}{4}} = \frac{\frac{\sqrt{3}}{3} + 1}{1 - \frac{\sqrt{3}}{3}}$

$\frac{3\left(\frac{\sqrt{3}}{3} + 1\right)}{3\left(1 - \frac{\sqrt{3}}{3}\right)} = \frac{(\sqrt{3}+3)(3+\sqrt{3})}{(3-\sqrt{3})(3+\sqrt{3})} = \frac{12+6\sqrt{3}}{6} = 2+\sqrt{3} = \tan\frac{5\pi}{12}$

Find the exact value of the sine, cosine, and tangent of the angle by using the half angle formulas.

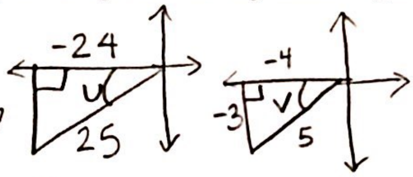
4.) $\frac{5\pi}{12}$ Hint: $\frac{\overset{u}{\left(\frac{5\pi}{6}\right)}}{2} = \frac{5\pi}{12}$

$\sin\frac{5\pi}{12} = \sin\left(\frac{\frac{5\pi}{6}}{2}\right) = +\sqrt{\frac{1-\cos\frac{5\pi}{6}}{2}} = \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{\frac{2+\sqrt{3}}{2}}{2}} = \sqrt{\frac{2+\sqrt{3}}{4}} = \frac{\sqrt{2+\sqrt{3}}}{2}$

$\cos\frac{5\pi}{12} = \cos\left(\frac{\frac{5\pi}{6}}{2}\right) = +\sqrt{\frac{1+\cos\frac{5\pi}{6}}{2}} = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{\frac{2-\sqrt{3}}{2}}{2}} = \sqrt{\frac{2-\sqrt{3}}{4}} = \frac{\sqrt{2-\sqrt{3}}}{2}$

$\tan\frac{5\pi}{12} = \tan\left(\frac{\frac{5\pi}{6}}{2}\right) = \frac{1-\cos\frac{5\pi}{6}}{\sin\frac{5\pi}{6}} = \frac{1+\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\frac{2+\sqrt{3}}{2}}{1/2} = 2+\sqrt{3}$

Find the exact value of the trigonometric function given that $\sin u = -\frac{7}{25}$ and $\cos v = -\frac{4}{5}$. (Both are in Quadrant III.)



5.) $\boxed{\cos(u+v) = \frac{3}{5}}$

cos u cos v − sin u sin v

$\left(-\frac{24}{25}\right)\left(-\frac{4}{5}\right) - \left(-\frac{7}{25}\right)\left(-\frac{3}{5}\right)$

$\frac{96}{125} - \frac{21}{125} = \frac{75}{125} = \frac{3}{5}$

6.) $\cos(u-v) = \frac{117}{125}$

cos u cos v + sin u sin v

$\left(-\frac{24}{25}\right)\left(-\frac{4}{5}\right) + \left(-\frac{7}{25}\right)\left(-\frac{3}{5}\right)$

$\frac{96}{125} + \frac{21}{125} = \frac{117}{125}$

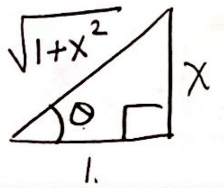
Write the trigonometric expression as an algebraic expression.

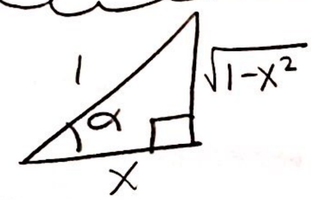
7.) $\cos(\arccos x - \arctan x)$

$\cos(\arccos x)\cos(\arctan x) + \sin(\arccos x)\sin(\arctan x)$

$x\left(\frac{1}{\sqrt{1+x^2}}\right) + \sqrt{1-x^2}\left(\frac{x}{\sqrt{1+x^2}}\right)$

$\frac{x + x\sqrt{1-x^2}}{\sqrt{1+x^2}}$





Simplify the expression.

8.) $\sin\left(\frac{3\pi}{2} + \theta\right)$

$\sin\frac{3\pi}{2}\cos\theta + \cos\frac{3\pi}{2}\sin\theta$

$-\cos\theta + 0(\sin\theta)$

$-\cos\theta$

9.) $\tan(\pi + \theta)$

$\frac{\tan\pi + \tan\theta}{1 - \tan\pi\tan\theta}$

$\frac{0 + \tan\theta}{1 - 0(\tan\theta)} = \tan\theta$

Why does this answer make sense?

(Think about the period of tangent

Find all solutions of the equation in the interval $[0, 2\pi)$.

10.) $\sin\left(x+\frac{\pi}{6}\right)-\sin\left(x-\frac{\pi}{6}\right)=\frac{1}{2}$

$$\left(\cancel{\sin x\cos\tfrac{\pi}{6}}+\cos x\sin\tfrac{\pi}{6}\right)-\left(\cancel{\sin x\cos\tfrac{\pi}{6}}-\cos x\sin\tfrac{\pi}{6}\right)=\tfrac{1}{2}$$

$$2\cos x\sin\tfrac{\pi}{6}=\tfrac{1}{2}$$

$$2\cos x\,(1/2)=\tfrac{1}{2}$$

$$\cos x=\tfrac{1}{2}$$

$$x=\left\{\tfrac{\pi}{3},\ \tfrac{5\pi}{3}\right\}$$

Find all solutions of the equation in the interval $[0, 2\pi)$.

11.) $\tan(x+\pi)-\cos\left(x-\frac{\pi}{2}\right)=0$

$\tan\pi=0$

$\cos\frac{\pi}{2}=0$

$\sin\frac{\pi}{2}=1$

$$\left(\frac{\tan x+\tan\pi}{1-\tan x\tan\pi}\right)-\left(\cos x\cos\tfrac{\pi}{2}+\sin x\sin\tfrac{\pi}{2}\right)=0$$

$$\tan x-\sin x=0$$

$$\frac{\sin x}{\cos x}-\sin x=0$$

$$\sin x\left(\frac{1}{\cos x}-1\right)=0$$

$\sin x=0$

$x=0,\pi$

$\sec x=1$

$\cos x=1$

$x=0$

$\boxed{\{0,\pi\}}$

12.) $\sin 2x-\sin x=0$

$$2\sin x\cos x-\sin x=0$$

$$\sin x(2\cos x-1)=0$$

$\sin x=0$ $\quad 2\cos x-1=0$

$x=0,\pi$ $\quad \cos x=1/2$

$x=\pi/3,\ 5\pi/3$

$$\boxed{\left\{0,\tfrac{\pi}{3},\pi,\tfrac{5\pi}{3}\right\}}$$

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Use a double-angle formula to rewrite the expression.

13.) $6\sin x\cos x$

$3(2\sin x\cos x)$

$3\sin 2x$

14.) $4-8\sin^2 x$

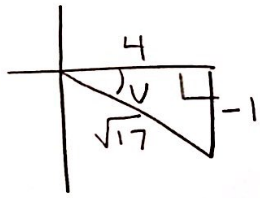
$4(1-2\sin^2 x)$

$4\cos 2x$

Using double-angle formulas, find the exact values of $\sin 2u$, $\cos 2u$, *and* $\tan 2u$.

15.) $\cot u = -4$; $\quad \overset{Q\,IV}{\dfrac{3\pi}{2} < u < 2\pi}$

$\cot u = -4 \qquad \tan u = -\frac{1}{4}$



$\sin 2u = 2\sin u\cos u = 2\left(-\frac{1}{\sqrt{17}}\right)\left(\frac{4}{\sqrt{17}}\right) = \frac{-8}{17}$

$\cos 2u = \cos^2 u - \sin^2 u = \left(\frac{4}{\sqrt{17}}\right)^2 - \left(-\frac{1}{\sqrt{17}}\right)^2 = \frac{16}{17} - \frac{1}{17} = \frac{15}{17}$

$\tan 2u = \dfrac{2\tan u}{1-\tan^2 u} = \dfrac{2(-1/4)}{1-(-1/4)^2} = \dfrac{-1/2}{1-1/16} = \dfrac{-1/2}{15/16} = \dfrac{-8/16}{15/16} = -8/15$

Find all solutions of the equation in the interval $[0, 2\pi)$.

16.) $\sin\dfrac{x}{2} + \cos x = 0$ $\qquad x = \pi$

$\pm\sqrt{\dfrac{1-\cos x}{2}} = -\cos x$

$\left(\pm\sqrt{\dfrac{1-\cos x}{2}}\right)^2 = (-\cos x)^2$

$\dfrac{1-\cos x}{2} = \cos^2 x$

$1-\cos x = 2\cos^2 x$

$2\cos^2 x + \cos x - 1 = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

$\cos x = 1/2 \qquad \cos x = -1$

$x = \left\{\cancel{\frac{\pi}{3}}, \cancel{\frac{5\pi}{3}}, \pi\right\}$

Use the product-to-sum formulas to write each product as a sum or difference.

) $\cos 4\theta \sin 6\theta$

$\cos 4\theta \sin 6\theta = \frac{1}{2}[\sin(4\theta+6\theta) - \sin(4\theta-6\theta)]$

$= \frac{1}{2}(\sin 10\theta - \sin(-2\theta))$

$= \boxed{\frac{1}{2}(\sin 10\theta + \sin 2\theta)}$

Use the sum-to-product formulas to write each sum or difference as a product.

18.) $\sin(\underset{u}{x+y}) - \sin(\underset{v}{x-y}) = 2\cos\left(\frac{x+y+x-y}{2}\right)\sin\left(\frac{x+y-(x-y)}{2}\right)$

$= 2\cos x \sin y$

Verify the identity.

19.) $\csc 2\theta = \dfrac{\csc\theta}{2\cos\theta}$

$\dfrac{1}{\sin 2\theta} =$

$\dfrac{1}{2\sin\theta\cos\theta} =$

$\dfrac{1}{\sin\theta} \cdot \dfrac{1}{2\cos\theta}$

$\dfrac{\csc\theta}{2\cos\theta} = \dfrac{\csc\theta}{2\cos\theta}$ ✓

20.) $\cos^4\theta - \sin^4\theta = \cos 2\theta$

$(\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta) =$

$1\ (\cos^2\theta - \sin^2\theta) =$

$\cos 2\theta = \cos 2\theta$ ✓

21.) $(\sin x + \cos x)^2 = 1 + \sin 2x$

$(\sin x + \cos x)(\sin x + \cos x)$

$\sin^2 x + 2\sin x\cos x + \cos^2 x =$

$1 + 2\sin x\cos x =$

$1 + \sin 2x = 1 + \sin 2x$ ✓

Find all solutions of the equation in the interval $[0, 2\pi)$.

22.) $\cos 4x + \cos 6x = 0$ $\left\{\frac{\pi}{10}, \frac{\pi}{2}, \frac{9\pi}{10}, \frac{13\pi}{10}, \frac{17\pi}{10}, \frac{3\pi}{10}, \frac{7\pi}{10}, \frac{11\pi}{10}, \frac{3\pi}{2}, \frac{19\pi}{10}\right\}$

$2\cos\left(\frac{4x+6x}{2}\right)\cos\left(\frac{4x-6x}{2}\right)=0$

$2\cos 5x \cos(-x) = 0$

$2\cos 5x \cos x = 0$

$\cos 5x = 0 \qquad \cos x = 0$

$5x = \frac{\pi}{2} + 2n\pi \qquad x = \frac{\pi}{2} + 2n\pi$

$5x = \frac{3\pi}{2} + 2n\pi \qquad x = \frac{3\pi}{2} + 2n\pi$

$x = \frac{\pi}{10} + \frac{2}{5}n\pi, \; \frac{3\pi}{10} + \frac{2}{5}n\pi$

23.) $\sin x + \sin 3x = 0$

$2\sin 2x \cos(-x) = 0$

$2\sin 2x \cos x = 0$

$\sin 2x = 0 \qquad \cos x = 0$

$2x = 0 + 2n\pi$

$2x = \pi + 2n\pi$

$x = \frac{\pi}{2}, \frac{3\pi}{2}, 0, \pi$